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A LANGUAGE OF SPACE

BY C. H. HINTON

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The following pages contain an outline of a proposed language of space, and indicate the uses and applications of such a language.

While written immediately as an addition to a book—"The Fourth Dimension," Second Edition, 1906, Appendix II.—on a special province of space-thinking, they present the subject of a space language in a sufficiently clear manner to obviate the necessity of a separate presentation, and will I hope lead to the adoption of a recognised nomenclature.

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1906

A LANGUAGE OF SPACE

The mere naming the parts of the figures we consider involves a certain amount of time and attention. This time and attention leads to no result, for with each new figure the nomenclature applied is completely changed, every letter or symbol is used in a different significance.

Surely it must be possible in some way to utilise the labour thus at present wasted!

Why should we not make a language for space itself, so that every position we want to refer to would have its own name? Then every time we named a figure in order to demonstrate its properties we should be exercising ourselves in the vocabulary of place.

If we use a definite system of names, and always refer to the same space position by the same name, we create as it were a multitude of little hands, each prepared to grasp a special point, position, or element, and hold it for us in its proper relations.

We make, to use another analogy, a kind of mental paper, which has somewhat of the properties of a sensitive plate, in that it will register, without effort, complex, visual, or tactual impressions.

But of far more importance than the applications of a space language to the plane and to solid space is the

facilitation it brings with it to the study of four-dimensional shapes.

I have delayed introducing a space language because all the systems I made turned out, after giving them a fair trial, to be intolerable. I have now come upon one which seems to present features of permanence, and I will here give an outline of it, so that it can be applied to the subject of the text, and in order that it may be subjected to criticism.

The principle on which the language is constructed is to sacrifice every other consideration for brevity.

It is indeed curious that we are able to talk and converse on every subject of thought except the fundamental one of space. The only way of speaking about the spatial configurations that underlie every subject of discursive thought is a co-ordinate system of numbers. This is so awkward and incommodious that it is never used. In thinking also, in realising shapes, we do not use it; we confine ourselves to a direct visualisation.

Now, the use of words corresponds to the storing up of our experience in a definite brain structure. A child, in the endless tactual, visual, mental manipulations it makes for itself, is best left to itself, but in the course of instruction the introduction of space names would make the teachers work more cumulative, and the child's knowledge more social.

Their full use can only be appreciated, if they are introduced early in the course of education; but in a minor degree any one can convince himself of their utility, especially in our immediate subject of handling four-dimensional shapes. The sum total of the results obtained in the preceding pages can be compendiously and accurately expressed in nine words of the Space Language.

In one of Plato's dialogues Socrates makes an experiment on a slave boy standing by. He makes certain perceptions of space awake in the mind of Meno's slave by directing his close attention on some simple facts of geometry.

By means of a few words and some simple forms we can repeat Plato's experiment on new ground.

Do we by directing our close attention on the facts of four dimensions awaken a latent faculty in ourselves? The old experiment of Plato's, it seems to me, has come down to us as novel as on the day he incepted it, and its significance not better understood through all the discussion of which it has been the subject.

Imagine a voiceless people living in a region where everything had a velvety surface, and who were thus deprived of all opportunity of experiencing what sound is. They could observe the slow pulsations of the air caused by their movements, and arguing from analogy, they would no doubt infer that more rapid vibrations were possible. From the theoretical side they could determine all about these more rapid vibrations. They merely differ, they would say, from slower ones, by the number that occur in a given time; there is a merely formal difference.

But suppose they were to take the trouble, go to the pains of producing these more rapid vibrations, then a totally new sensation would fall on their rudimentary ears. Probably at first they would only be dimly conscious of Sound, but even from the first they would become aware that a merely formal difference, a mere difference in point of number in this particular respect, made a great difference practically, as related to them. And to us the difference between three and four dimensions is merely formal, numerical. We can tell formally all about four dimensions, calculate the relations that would exist. But that the difference is merely formal does not prove that it is a futile and empty task, to present to ourselves as closely as we can the phenomena of four dimensions. In our formal

knowledge of it, the whole question of its actual relation to us, as we are, is left in abeyance.

Possibly a new apprehension of nature may come to us through the practical, as distinguished from the mathematical and formal, study of four dimensions. As a child handles and examines the objects with which he comes in contact, so we can mentally handle and examine four-dimensional objects. The point to be determined is this. Do we find something cognate and natural to our faculties, or are we merely building up an artificial presentation of a scheme only formally possible, conceivable, but which has no real connection with any existing or possible experience?

This, it seems to me, is a question which can only be settled by actually trying. This practical attempt is the logical and direct continuation of the experiment Plato devised in the "Meno."

Why do we think true? Why, by our processes of thought, can we predict what will happen, and correctly conjecture the constitution of the things around us? This is a problem which every modern philosopher has considered, and of which Descartes, Leibnitz, Kant, to name a few, have given memorable solutions. Plato was the first to suggest it. And as he had the unique position of being the first devisor of the problem, so his solution is the most unique. Later philosophers have talked about consciousness and its laws, sensations, categories. But Plato never used such words. Consciousness apart from a conscious being meant nothing to him. His was always an objective search. He made man's intuitions the basis of a new kind of natural history.

In a few simple words Plato puts us in an attitude with regard to psychic phenomena—the mind—the ego—"what we are," which is analogous to the attitude scientific men of the present day have with regard to the phenomena of outward nature. Behind this first apprehension of ours of nature, there is an infinite depth to be learned and known. Plato said that behind the phenomena of mind that Meno's slave boy exhibited, there was a vast, an infinite perspective. And his singularity, his originality, comes out most strongly marked in this, that the perspective, the complex phenomena beyond were, according to him, phenomena of personal experience. A footprint in the sand means a man to a being that has the conception of a man. But to a creature that has no such conception, it means a curious mark, somehow resulting from the concatenation of ordinary occurrences. Such a being would attempt merely to explain how causes known to him could so coincide as to produce such a result; he would not recognise its significance.

Plato introduced the conception which made a new kind of natural history possible. He said that Meno's slave boy thought true about things he had never learned, because his "soul" had experience. I know this will sound absurd to some people, and it flies straight in the face of the maxim, that explanation consists in showing how an effect depends on simple causes. But what a mistaken maxim that is! Can any single instance be shown of a simple cause? Take the behaviour of spheres for instance; say those ivory spheres, billiard balls, for example. We can explain their behaviour by supposing they are homogeneous elastic solids. We can give formulæ which will account for their movements in every variety. But are they homogeneous elastic solids? No, certainly not. They are complex in physical and molecular structure, and atoms and ions beyond open an endless vista. Our simple explanation is false, false as it can be. The balls act as if they were homogeneous elastic spheres. There is a statistical simplicity in the resultant of very complex conditions, which makes that artificial conception useful.

But its usefulness must not blind us to the fact that it is artificial. If we really look deep into nature, we find a much greater complexity than we at first suspect. And so behind this simple "I," this myself, is there not a parallel complexity? Plato's "soul" would be quite acceptable to a large class of thinkers, if by "soul" and the complexity he attributes to it, he meant the product of a long course of evolutionary changes, whereby simple forms of living matter endowed with rudimentary sensation had gradually developed into fully conscious beings.

But Plato does not mean by "soul" a being of such a kind. His soul is a being whose faculties are clogged by its bodily environment, or at least hampered by the difficulty of directing its bodily frame—a being which is essentially higher than the account it gives of itself through its organs. At the same time Plato's soul is not incorporeal. It is a real being with a real experience. The question of whether Plato had the conception of non-spatial existence has been much discussed. The verdict is, I believe, that even his "ideas" were conceived by him as beings in space, or, as we should say, real. Plato's attitude is that of Science, inasmuch as he thinks of a world in Space. But, granting this, it cannot be denied that there is a fundamental divergence between Plato's conception and the evolutionary theory, and also an absolute divergence between his conception and the genetic account of the origin of the human faculties. The functions and capacities of Plato's "soul" are not derived by the interaction of the body and its environment.

Plato was engaged on a variety of problems, and his religious and ethical thoughts were so keen and fertile that the experimental investigation of his soul appears involved with many other motives. In one passage Plato will combine matter of thought of all kinds and from all sources, overlapping, interrunning. And in no case is he more involved and rich than in this question of the soul. In fact, I wish there were two words, one denoting that being, corporeal and real, but with higher faculties than we manifest in our bodily actions, which is to be taken as the subject of experimental investigation; and the other word denoting "soul" in the sense in which it is made the recipient and the promise of so much that men desire. It is the soul in the former sense that I wish to investigate, and in a limited sphere only. I wish to find out, in continuation of the experiment in the Meno, what the "soul" in us thinks about extension, experimenting on the grounds laid down by Plato. He made, to state the matter briefly, the hypothesis with regard to the thinking power of a being in us, a "soul." This soul is not accessible to observation by sight or touch, but it can be observed by its functions; it is the object of a new kind of natural history, the materials for constructing which lie in what it is natural to us to think. With Plato "thought" was a very wide-reaching term, but still I would claim in his general plan of procedure a place for the particular question of extension.

The problem comes to be, "What is it natural to us to think about matter *qua* extended?"

First of all, I find that the ordinary intuition of any simple object is extremely imperfect. Take a block of differently marked cubes, for instance, and become acquainted with them in their positions. You may think you know them quite well, but when you turn them round —rotate the block round a diagonal, for instance—you will find that you have lost track of the individuals in their new positions. You can mentally construct the block in its new position, by a rule, by taking the remembered sequences, but you don't know it intuitively. By observation of a block of cubes in various positions, and very expeditiously by a use of Space names applied to the

cubes in their different presentations, it is possible to get an intuitive knowledge of the block of cubes, which is not disturbed by any displacement. Now, with regard to this intuition, we moderns would say that I had formed it by my tactual visual experiences (aided by hereditary predisposition). Plato would say that the soul had been stimulated to recognise an instance of shape which it knew. Plato would consider the operation of learning merely as a stimulus; we as completely accounting for the result. The latter is the more common-sense view. But, on the other hand, it presupposes the generation of experience from physical changes. The world of sentient experience, according to the modern view, is closed and limited; only the physical world is ample and large and of ever-to-be-discovered complexity. Plato's world of soul, on the other hand, is at least as large and ample as the world of things.

Let us now try a crucial experiment. Can I form an intuition of a four-dimensional object? Such an object is not given in the physical range of my sense contacts. All I can do is to present to myself the sequences of solids, which would mean the presentation to me under my conditions of a four-dimensional object. All I can do is to visualise and tactualise different series of solids which are alternative sets of sectional views of a four-dimensional shape.

If now, on presenting these sequences, I find a power in me of intuitively passing from one of these sets of sequences to another, of, being given one, intuitively constructing another, not using a rule, but directly apprehending it, then I have found a new fact about my soul, that it has a four-dimensional experience; I have observed it by a function it has.

I do not like to speak positively, for I might occasion a loss of time on the part of others, if, as may very well be, I am mistaken. But for my own part, I think there are indications of such an intuition; from the results of my experiments, I adopt the hypothesis that that which thinks in us has an ample experience, of which the intuitions we use in dealing with the world of real objects are a part; of which experience, the intuition of four-dimensional forms and motions is also a part. The process we are engaged in intellectually is the reading the obscure signals of our nerves into a world of reality, by means of intuitions derived from the inner experience.

The image I form is as follows. Imagine the captain of a modern battle-ship directing its course. He has his charts before him; he is in communication with his associates and subordinates; can convey his messages and commands to every part of the ship, and receive information from the conning-tower and the engine-room. Now suppose the captain immersed in the problem of the navigation of his ship over the ocean, to have so absorbed himself in the problem of the direction of his craft over the plane surface of the sea that he forgets himself. All that occupies his attention is the kind of movement that his ship makes. The operations by which that movement is produced have sunk below the threshold of his consciousness, his own actions, by which he pushes the buttons, gives the orders, are so familiar as to be automatic, his mind is on the motion of the ship as a whole. In such a case we can imagine that he identifies himself with his ship; all that enters his conscious thought is the direction of its movement over the plane surface of the ocean.

Such is the relation, as I imagine it, of the soul to the body. A relation which we can imagine as existing momentarily in the case of the captain is the normal one in the case of the soul with its craft. As the captain is capable of a kind of movement, an amplitude of motion, which does not enter into his thoughts with regard to the directing the ship over the plane surface of the ocean, so the soul is capable of a kind of movement, has an amplitude of motion, which is not used in its task of directing the body in the three-dimensional region in which the body's activity lies. If for any reason it became necessary for the captain to consider three-dimensional motions with regard to his ship, it would not be difficult for him to gain the materials for thinking about such motions; all he has to do is to call his own intimate experience into play. As far as the navigation of the ship, however, is concerned, he is not obliged to call on such experience. The ship as a whole simply moves on a surface. The problem of three-dimensional movement does not ordinarily concern its steering. And thus with regard to ourselves all those movements and activities which characterise our bodily organs are three-dimensional; we never need to consider the ampler movements. But we do more than use the movements of our body to effect our aims by direct means; we have now come to the pass when we act indirectly on nature, when we call processes into play which lie beyond the reach of any explanation we can give by the kind of thought which has been sufficient for the steering of our craft as a whole. When we come to the problem of what goes on in the minute, and apply ourselves to the mechanism of the minute, we find our habitual conceptions inadequate.

The captain in us must wake up to his own intimate nature, realise those functions of movement which are his own, and in virtue of his knowledge of them apprehend how to deal with the problems he has come to.

Think of the history of man. When has there been a time, in which his thoughts of form and movement were not exclusively of such varieties as were adapted for his bodily performance? We have never had a demand to conceive what our own most intimate powers are. But, just as little as by immersing himself in the steering of his ship over the plane surface of the ocean, a captain can loose the faculty of thinking about what he actually does, so little can the soul loose its own nature. It can be roused to an intuition that is not derived from the experience which the senses give. All that is necessary is to present some few of those appearances which, while inconsistent with three-dimensional matter, are yet consistent with our formal knowledge of four-dimensional matter, in order for the soul to wake up and not begin to learn, but of its own intimate feeling fill up the gaps in the presentiment, grasp the full orb of possibilities from the isolated points presented to it. In relation to this question of our perceptions, let me suggest another illustration, not taking it too seriously, only propounding it to exhibit the possibilities in a broad and general way.

In the heavens, amongst the multitude of stars, there are some which, when the telescope is directed on them, seem not to be single stars, but to be split up into two. Regarding these twin stars through a spectroscope, an astronomer sees in each a spectrum of bands of colour and black lines. Comparing these spectrums with one another, he finds that there is a slight relative shifting of the dark lines, and from that shifting he knows that the stars are rotating round one another, and can tell their relative velocity with regard to the earth. By means of his terrestrial physics he reads this signal of the skies. This shifting of lines, the mere slight variation of a black line in a spectrum, is very unlike that which the astronomer knows it means. But it is probably much more like what it means than the signals which the nerves deliver are like the phenomena of the outer world.

No picture of an object is conveyed through the nerves. No picture of motion, in the sense in which we postulate its existence, is conveyed through the nerves. The actual

deliverances of which our consciousness takes account are probably identical for eye and ear, sight and touch.

If for a moment I take the whole earth together and regard it as a sentient being, I find that the problem of its apprehension is a very complex one, and involves a long series of personal and physical events. Similarly the problem of our apprehension is a very complex one. I only use this illustration to exhibit my meaning. It has this especial merit, that, as the process of conscious apprehension takes place in our case in the minute, so, with regard to this earth being, the corresponding process takes place in what is relatively to it very minute.

Now, Plato's view of a soul leads us to the hypothesis that that which we designate as an act of apprehension may be a very complex event, both physically and personally. He does not seek to explain what an intuition is; he makes it a basis from whence he sets out on a voyage of discovery. Knowledge means knowledge; he puts conscious being to account for conscious being. He makes an hypothesis of the kind that is so fertile in physical science—an hypothesis making no claim to finality, which marks out a vista of possible determination behind determination, like the hypothesis of space itself, the type of serviceable hypotheses.

And, above all, Plato's hypothesis is conducive to experiment. He gives the perspective in which real objects can be determined; and, in our present enquiry, we are making the simplest of all possible experiments—we are enquiring what it is natural to the soul to think of matter as extended.

Aristotle says we always use a " phantasm " in thinking, a phantasm of our corporeal senses a visualisation or a tactualisation. But we can so modify that visualisation or tactualisation that it represents something not known by the senses. Do we by that representation wake up an intuition of the soul? Can we by the presentation of these hypothetical forms, that are the subject of our present discussion, wake ourselves up to higher intuitions? And can we explain the world around by a motion that we only know by our souls?

Apart from all speculation, however, it seems to me that the interest of these four-dimensional shapes and motions is sufficient reason for studying them, and that they are the way by which we can grow into a fuller apprehension of the world as a concrete whole.

Space Names.

If the words written in the squares drawn in fig. 1 are used as the names of the squares in the positions in which they are placed, it is evident that a combination of these names will denote a figure composed of the designated squares. It is found to be most convenient to take as the initial square that marked with an asterisk, so that the directions of progression are towards the observer and to his right. The directions of progression, however, are arbitrary, and can be chosen at will.

* en	et	el
an	at	al
in	it	il

Fig. 1.

Thus *et*, *at*, *it*, *an*, *al* will denote a figure in the form of a cross composed of five squares.

Here, by means of the double sequence, *e*, *a*, *i* and *n*, *t*, *l*, it is possible to name a limited collection of space elements.

The system can obviously be extended by using letter sequences of more members.

But, without introducing such a complexity, the principles of a space language can be exhibited, and a nomenclature obtained adequate to all the considerations of the preceding pages.

1. *Extension.*

Call the large squares in fig. 2 by the name written in them. It is evident that each can be divided as shown in fig. 1. Then the small square marked 1 will be "en" in "En," or "Enen." The square marked 2 will be "et" in "En" or "Enet," while the square marked 4 will be "en" in "Et" or "Eten." Thus the square 5 will be called "Ilil."

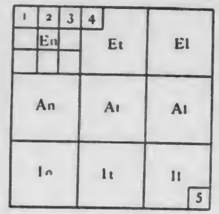


Fig. 2.

This principle of extension can be applied in any number of dimensions.

2. *Application to Three-Dimensional Space.*

To name a three-dimensional collocation of cubes take the upward direction first, secondly the direction towards the observer, thirdly the direction to his right hand.

These form a word in which the first letter gives the place of the cube upwards, the second letter its place towards the observer, the third letter its place to the right.

len	let	lel
lan	lat	lal
lin	lit	lil

ten	tet	tel
tan	tat	tal
tin	tit	til

* nen	net	nel
nan	nat	nal
nin	nit	nil

We have thus the following scheme, which represents the set of cubes of column 1, fig. 101, page 165.

We begin with the remote lowest cube at the left hand, where the asterisk is placed (this proves to be by far the most convenient origin to take for the normal system).

Thus "nen" is a "null" cube, "ten" a red cube on it, and "len" a "null" cube above "ten."

By using a more extended sequence of consonants and vowels a larger set of cubes can be named.

To name a four-dimensional block of tesseracts it is simply necessary to prefix an "e," an "a," or an "i" to the cube names.

Thus the tesseract blocks schematically represented on page 165, fig. 101 are named as follows:—

elen	elet	elel	alen	alet	alel	ilen	ilet	ilel
elan	elat	elal	alan	alat	alal	ilan	ilat	ilal
elin	elit	elil	alin	alit	alil	ilin	ilit	ilil

eten	etet	etel	aten	atet	atel	iten	itet	itel
etan	etat	etal	atan	atat	atal	itan	itat	ital
etin	etit	etil	atin	atit	atil	itin	itit	itil

*

enen	enet	enel	anen	anet	anel	inen	inet	inel
enan	enat	enal	anan	anat	anal	inan	inat	inal
enin	enit	enil	anin	anit	anil	inin	init	inil

1 2 3

2. Derivation of Point, Line, Face, etc., Names.

The principle of derivation can be shown as follows: Taking the square of squares

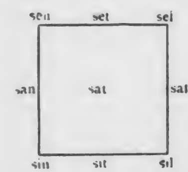
en	et	el
an	at	al
in	it	il

the number of squares in it can be enlarged and the whole kept the same size.

en	et	et	el
an	at	at	al
an	at	at	al
in	it	it	il

Compare fig. 79, p. 138, for instance, or the bottom layer of fig. 84.

Now use an initial "s" to denote the result of carrying this process on to a great extent, and we obtain the limit names, that is the point, line, area names for a square. "Sat" is the whole interior. The corners are "sen," "sel," "sin," "sil," while the lines are "san," "sal," "set," "sit."



I find that by the use of the initial "s" these names come to be practically entirely disconnected with the systematic names for the square from which they are derived. They are easy to learn, and when learned can be used readily with the axes running in any direction.

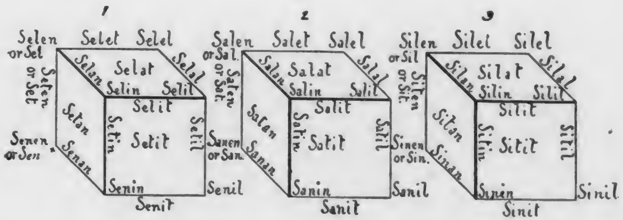
To derive the limit names for a four-dimensional rectangular figure, like the tesseract, is a simple extension of this process. These point, line, etc., names include those which apply to a cube, as will be evident on inspection of the first cube of the diagrams which follow.

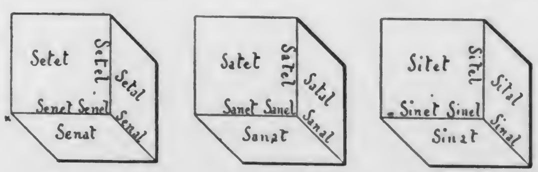
All that is necessary is to place an "s" before each of the names given for a tesseract block. We then obtain apellatives which, like the colour names on page 174, fig. 103, apply to all the points, lines, faces, solids, and to the hypersolid of the tesseract. These names have the advantage over the colour marks that each point, line, etc., has its own individual name.

In the diagrams I give the names corresponding to the positions shown in the coloured plate or described on p. 174. By comparing cubes 1, 2, 3 with the first row of cubes in the coloured plate, the systematic names of each of the points, lines, faces, etc., can be determined. The asterisk shows the origin from which the names run.

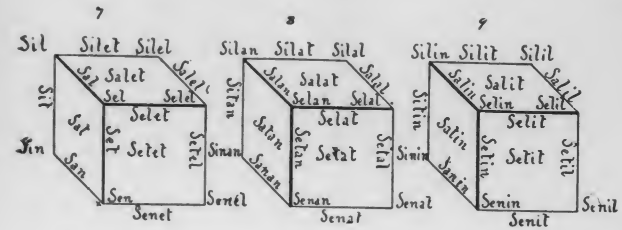
These point, line, face, etc., names should be used in connection with the corresponding colours. The names should call up coloured images of the parts named in their right connection.

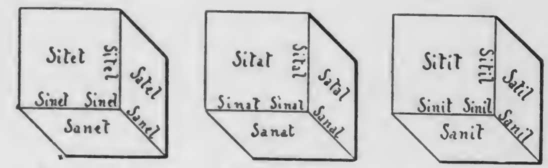
It is found that a certain abbreviation adds vividness of distinction to these names. If the final "en" be dropped wherever it occurs the system is improved. Thus instead of "senen," "seten," "selen," it is preferable to abbreviate to "sen," "set," "sel," and also use "san," "sin" for "sanen," "sinen."





Interior Setet Interior Satat Interior Sitat





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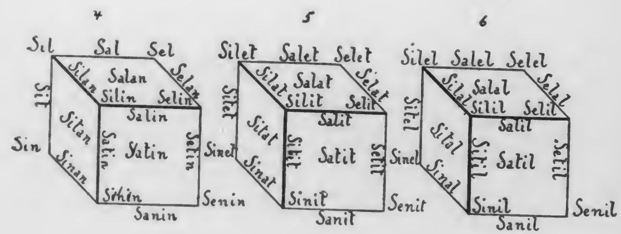

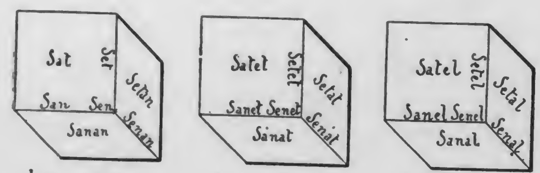

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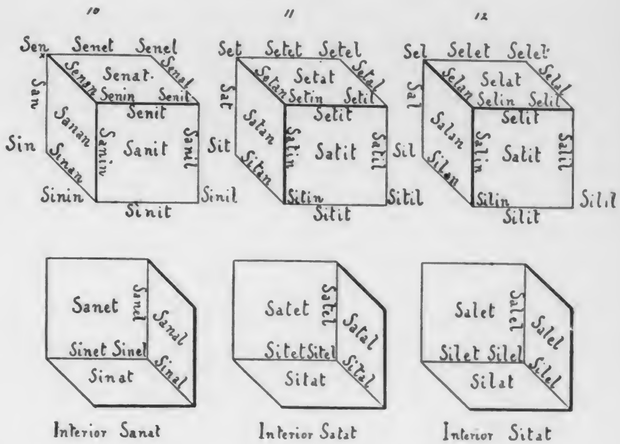

We can now name any section. Take *e.g.* the line in the first cube from senin to senel, we should call the line running from senin to senel, senin senat senel, a line light yellow in colour with null points.

Here senat is the name for all of the line except its ends. Using "senat" in this way does not mean that the line is the whole of senat, but what there is of it is senat. It is a part of the senat region. Thus also the triangle, which has its three vertices in senin, senel, selen, is named thus:

Area : setat.
Sides : setan, senat, setet.
Vertices : senin, senel, sel.

The tetrahedron section of the tesseract can be thought of as a series of plane sections in the successive sections of the tesseract shown in fig. 114, p. 191. In b_0 the section is the one written above. In b_1 the section is made by a

plane which cuts the three edges from sanen intermediate of their lengths and thus will be:

Area : satat.
Sides : satan, sanat, satet.
Vertices : sanan, sanet, sat.

The sections in b_2, b_3 will be like the section in b_1 but smaller.

Finally in b_4 the section plane simply passes through the corner named sin.

Hence, putting these sections together in their right relation, from the face setat, surrounded by the lines and points mentioned above, there run:

3 faces : satan, sanat, satet
3 lines : sanan, sanet, sat

and these faces and lines run to the point sin. Thus the tetrahedron is completely named.

The octahedron section of the tesseract, which can be traced from fig. 72, p. 129 by extending the lines there drawn, is named:

Front triangle selin, selat, selel, setal, senil, setit, selin with area setat.

The sections between the front and rear triangle, of which one is shown in 1b another in 2b, are thus named, points and lines, salan, salat, salet, satet, satel, satal, sanal, sanat, sanit, satit, satin, satan, salan.

The rear triangle found in 3b by producing lines is sil, sitet, sinel, sinat, sinin, sitan, sil.

The assemblage of sections constitute the solid body of the octahedron satat with triangular faces. The one from the line selat to the point sil, for instance, is named selin, selat, selel, salet, salat, salan, sil. The whole interior is salat.

Shapes can easily be cut out of cardboard which, when folded together, form not only the tetrahedron and the octohedron, but also samples of all the sections of the tesseract taken as it passes cornerwise through our space. To name and visualise with appropriate colours a series of these sections is an admirable exercise for obtaining familiarity with the subject.

Extension and Connection with Numbers.

By extending the letter sequence it is of course possible to name a larger field. By using the limit names the corners of each square can be named.

Thus "en sen," "an sen," etc., will be the names of the points nearest the origin in "en" and in "an."

A field of points of which each one is indefinitely small is given by the names written below.

ensen	etsen	elsen
ansen	atsen	alsen
insen	itsen	ilsen

The squares are shown in dotted lines, the names denote the points. These points are not mathematical points, but really minute areas.

Instead of starting with a set of squares and naming them, we can start with a set of points.

By an easily remembered convention we can give names to such a region of points.

Let the space names with a final "e" added denote the mathematical points at the corner of each square nearest the origin. We have then

	0	1	2
0	ene	ete	ele
1	ane	ate	ale
2	ine	ite	ile

for the set of mathematical points indicated. This system is really completely independent of the area system and is connected with it merely for the purpose of facilitating the memory processes. The word "ene" is pronounced like "eny," with just sufficient attention to the final vowel to distinguish it from the word "en."

Now, connecting the numbers 0, 1, 2 with the sequence e, a, i, and also with the sequence n, t, l, we have a set of points named as with numbers in a co-ordinate system. Thus "ene" is (0, 0) "ate" is (1, 1) "ite" is (2, 1). To pass to the area system the rule is that the name of the square is formed from the name of its point nearest to the origin by dropping the final e.

By using a notation analogous to the decimal system a larger field of points can be named. It remains to assign a letter sequence to the numbers from positive 0 to positive 9, and from negative 0 to negative 9, to obtain a system which can be used to denote both the usual co-ordinate system of mapping and a system of named squares. The names denoting the points all end with e. Those that denote squares end with a consonant.

There are many considerations which must be attended to in extending the sequences to be used, such as uniqueness in the meaning of the words formed, ease of pronunciation, avoidance of awkward combinations.

I drop "s" altogether from the consonant series and short "u" from the vowel series. It is convenient to have unsignificant letters at disposal. A double consonant like "st" for instance can be referred to without giving it a local significance by calling it "ust." I increase the number of vowels by considering a sound like "ra" to be a vowel, using, that is, the letter "r" as forming a compound vowel.

The series is as follows:—

CONSONANTS.

	0	1	2	3	4	5	6	7	8	9
positive	n	t	l	p	f	sh	k	ch	nt	st
negative	z	d	th	b	v	m	g	j	nd	sp

VOWELS.

	0	1	2	3	4	5	6	7	8	9
positive	e	a	i	ee	ae	ai	ar	ra	ri	ree
negative	er	o	oo	io	oe	iu	or	ro	roo	rio

Pronunciation.—e as in men; a as in man; i as in in; ee as in between; ae as ay in may; ai as i in mine; ar as in art; er as ear in earth; o as in on; oo as oo in soon; io as in clarion; oe as oa in oat; iu pronounced like yew.

To name a point such as (23, 41) it is considered as (3, 1) on from (20, 40) and is called "ifeete." It is the initial point of the square ifeet of the area system.

The preceding amplification of a space language has been introduced merely for the sake of completeness. As has already been said nine words and their combinations, applied to a few simple models suffice for the purposes of our present enquiry.

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