HAY

ON

PROPORTION.

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OR THE GEOMETRIC PRINCIPLE OF BEAUTY, ANALYSED.

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INTRODUCTION.

To attempt to reduce to fixed principles what is called Taste in the combination of forms, is a difficult and perhaps a presumptuous task, although such principles are universally felt to exist. Yet it is not a hopeless labour, as it seems almost certain that the Grecians, at the period of their highest refinement, had a positive geometric principle of beauty systematically developed and applied in all their works.

It is recorded in a curious treatise called the "Beau Ideal," quoted by Hogarth in the preface to his "Analysis of Beauty," that a key for finding all harmonious proportions in Painting, Sculpture, Architecture, Music, &c., was brought to Greece by Pythagoras, on his return from travelling through Phœnicia, Egypt, and Chaldea, (about A.M. 3484,) and that its application to the arts raised them to their highest pitch of excellence. Flaxman, the celebrated sculptor, in remarking, in his lectures, that the lines of Grecian composition enchant the beholder by the harmony and perfection of their geometric beauty, adds—" This portion of study seems to have been highly esteemed by Pamphilus,* the learned Macedonian painter, who denied that any one could succeed in the study of painting without arithmetic and geometry. But this magic bond of arrangement was utterly lost when the other perfections of Grecian genius were overwhelmed in barbarism."

Vitruvius, the well-known Roman writer on architecture, seems also to have been impressed with a conviction of the fact, that the ancient Grecians possessed geometric rules of beauty, but at the same time appears in considerable doubt as to what these rules were. At the commencement of his work on Civil Architecture, he observes—" Symmetry results from proportion, which, in the Greek language, is termed Analogia. Proportion is the commensuration of the various constituent parts with the whole, in the existence of which symmetry

* Pamphilus flourished A.M. 3641. He was the master of Apelles, and is said to have applied the key of harmony, or analogy, of the Grecians to the art of Painting.

is found to consist." He then remarks, that the proportions of a perfect building ought to resemble those of a well-formed human body, of the proportions of which he gives various details, adding, that the general standard by which all measurements were made was deduced from its members.

He says, also, that the perfect number amongst the ancients was thought to be ten, but that mathematicians, on the contrary, contended that it was six; and upon this principle the cities of Greece appeared to have divided the drachma into six parts. The Romans, however, preferred the ancient number ten; but, in process of time, being led to consider both ten and six as perfect numbers, they formed what they thought a still more perfect one by combining the two.

That doubt existed as to the proper mode of harmonious division at the period he writes, is clear from what follows:—" If it be true," he adds, " that the decennary notation was suggested by the members (the digits, for example) of man, and that the laws of proportion arose from the relative measures existing between certain parts of each member and the whole body, it will follow, that those are entitled to our commendation, who, in building temples to their deities, proportioned the edifices so that the several parts might be commensurate with the whole."

This knowledge of the principles of geometric beauty which the ancient Grecians possessed, and applied systematically to the arts of design, seems, therefore, to have been lost before the time of Vitruvius.*

Although my former essay was almost exclusively directed to proving the analogy that exists in the harmonies of sound, colour, and form, and although I find that in a very extraordinary work,† lately published, it has been clearly proved that such an analogy not only exists as regards the elements of harmony, but also pervades nature, science, and art generally, I must support my present attempt by again referring to the same subject, because the effects of harmony in regard to sound have been thoroughly investigated as a branch of natural philosophy, and fixed principles established for their regulation.

In Painting and Sculpture, form addresses itself to the eye in two ways. The first quality required in works of these arts, and that

* In the era of Julius Cæsar.

† Field's Outlines of Analogical Philosophy. London: 1839.

which is most easily appreciated by the generality of mankind, is a faithful imitation of the configuration of any natural object or objects that may be represented in such works. These arts are consequently called the imitative arts, in contradistinction to those in which no natural object is imitated. The other quality of form required to constitute excellence in works of art, lies in what is technically called composition. This, in Painting and Sculpture, is the disposition or arrangement of such natural objects, or their parts, as may be brought together in forming a subject, independently of their individual merits. This disposition may be made so as greatly to enhance the beauty or grandeur of the subject, by the scientific combination of similarity and dissimilarity, and of various modes of simplicity and variety. No production in Painting or Sculpture can be reckoned a great work of art which depends exclusively on the first of those qualities of form; but many have been reckoned so from the science displayed in their mode of composition, although defective in accuracy of imitation.

These qualities are very different in their effect upon the mind. Imitation in works of art is appreciated by the degree of deception to which the judgment, through the limited capacity of the visual organ, is subjected; while scientific composition of form seems to be appreciated by an inherent feeling responsive to certain mathematical principles of propriety and harmony existing in nature, and conveying an impression to the mind through the medium of the senses.

The beauty of all original architectural compositions depends upon mathematical harmony alone, because in such there is no imitation; and it can scarcely be doubted that the Five Orders owe their origin and the perfection of their proportions to some systematic mode of applying those principles practically in this art. Upon this subject Field makes the following excellent observation:—" In proportion as we acquire rules and principles of our own, we shall be released from the servile necessity of continuing mere imitators of those ancients, with the philosophy of whose practice we are little acquainted, and who certainly did not work without principles themselves." *

So far as I know, Burke is the only writer on taste who does not admit that proportion is one of the constituents of beauty. He says that " proportion relates almost wholly to convenience ;"† and that it

* Outlines of Analogical Philosophy.

† Essay on the Sublime and Beautiful.

should therefore be considered as a creature of the understanding, rather than a primary cause acting on the senses and imagination. It is very evident that proportion and fitness are here, as well as throughout this part of the Essay, considered as the same quality; but they appear to be qualities of a very different nature. Fitness certainly is a creature of the understanding relating to use and convenience, and cannot therefore be considered a necessary concomitant of beauty. But proportion is a quality of a very different kind; it is the essence of symmetry, and symmetry is the first principle of harmony to the eye. Proportion must surely, therefore, be regarded as a constituent of beauty in form and figures.

In treating of this property in forms, apart from fitness, Burke defines it thus:—" Proportion is the measure of relative quantity Since all quantity is divisible, it is evident that every distinct part into which any quantity is divided must bear some relation to the other parts, or to the whole. These relations give an origin to the idea of proportion. They are discovered by mensuration, and they are the objects of mathematical enquiry. But whether any part of any determinate quantity be a fourth, or a fifth, or a sixth, or a moiety of the whole; or whether it be of equal length with any other part, or double its length, or but one half, is a matter merely indifferent to the mind." He then proceeds to state, that all proportions and arrangements of quantity are alike to the understanding; that beauty has nothing to do with calculation and geometry; and concludes, "If it had, we might then point out some certain measures which we could demonstrate to be beautiful, either as simply considered or as related to others."

In despite of so great an authority to the contrary, an attempt shall be made, in the present Essay, to prove that beauty *does* depend upon calculation and geometry; and that, therefore, we *can* point out and demonstrate certain measures to be beautiful, either as simply considered, or as related to others.

The beauty of forms must consist in the production of a pleasing effect upon the visual perceptions;* the physical effect of forms, when

* This fact is admitted by this eminent author in another part of the same Essay, where he observes,—"Another principle of beautiful objects is, that the line of their parts is continually varying its direction; but it varies it by a very insensible deviation—it never varies it so quickly as to surprise, or, by the sharpness of its angle, to cause any twitching or convulsion of the optic nerve."

pleasing to that sense, is appropriately called harmony, and when displeasing it is called discord. Now, when we find that combined or successive sounds are harmonious or discordant to the sense of hearing, agreeably to the arithmetical proportion they bear to one another in the number of pulsations they occasion in the surrounding atmosphere in a given period, and that these pulsations have an equally exact geometrical proportion relatively to the size of the vibratory body producing them, it appears extraordinary that so acute a reasoner as Burke should advance a doctrine so much at variance with the general tenor of his excellent Essay, especially as he admits, in another part, that "if we can comprehend clearly how things operate upon one of our senses, there can be very little difficulty in conceiving in what manner they affect the rest."

It is with much reluctance that I venture to differ so decidedly on this point from the opinions of so great an author, particularly as I cordially concur in the general principles developed in his admirable Essay on the Sublime and Beautiful; as well as in the various observations by which they are introduced, and in none more than in that with which I shall conclude this Introduction:—

"By looking into physical causes, our minds are opened and enlarged; and in this pursuit, whether we take or whether we lose our game, the chase is certainly of service."

PROPORTION,

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Proportion in its simplest mode is to form what time is to music, or measure to poetry;* but in its more complex mode, it is to form what grammar is to language, or harmony to music. Proportion may lie in the relative sizes of two or more objects—the relative dimensions that the length bears to the breadth of an object—the relative obtusity or acuteness of various angles—the relative degrees of cur-

* Poetry is composed of two things,—of the natural perception of the beautiful, and of the artistic development of this perception. In the former sense we are all poets; in the latter sense only a few possess the divine gift, and merit the distinguished name. We are all poets; for we are all capable of seizing, among the aspects of the actual, that harmony of proportions which constitutes beauty, and of finding in the field of the possible and the spiritual, that image of perfection of which external grace and sublimity are simply the embodiments. The meanest event, the most insignificant object, if suggestive to us of brighter thoughts and deeper feelings than those that people the range of our ordinary musings, become for us a poetical event—a poetical object.—*Maccall's Agents of Civilization.*

vature in various objects, or in the parts of one object, or, it may be, in the general relation that various forms bear to one another in rendering their combinations harmonious. Proportion is, in short, that geometrical quality in forms and figures by which they are rendered pleasing to the sense of sight, independently of their use or any other consideration. A short description of the structure of the organ of this sense, and the manner in which geometrical configuration acts upon it, will therefore be an appropriate commencement to this essay.

Fig. 1.

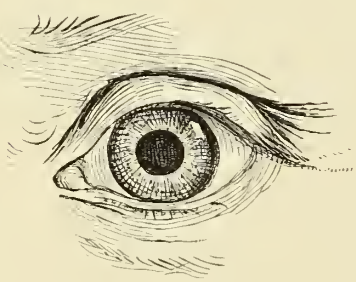
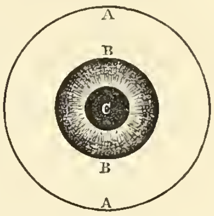


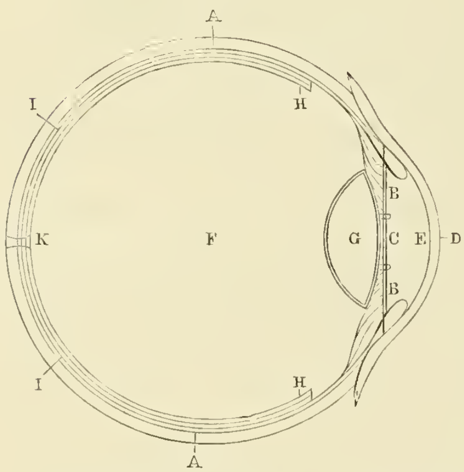
Figure 1 represents the external appearance of the human eye, the visible parts of which are the sclerotic coat, or white of the eye, to which the muscles for giving it motion are attached; the iris, or that part which denotes the colour of the eye, being a flat membrane of a circular form, having in its centre the pupil, apparently a dark circular spot, but in reality a simple aperture in the opaque membrane that surrounds it, its seeming depth of colour being derived from the darkness of the inner chamber of the eye. When this organ of sense is examined anatomically, it is found to be of a globular form, as shown in figure 2, on which A is the white or sclerotic coat, B the iris, and C the pupil. There is upon this globe a slight projection in front of the iris. The front covering of this projection is the cornea, a transparent circular membrane embracing the iris and pupil, and through which they are seen, as in figure 1. It closes in the anterior chamber of the eye, which contains a transparent liquid called the aqueous humour. Behind the iris is the posterior chamber of the eye, which is also filled with a transparent liquid, called the vitreous humour. Attached to the back of the iris, and close to the pupil, is that portion of the eye called the crystalline lens, which consists of concentric coats composed of fibres, and is contained within a transparent capsule or bag. Upon the inner surface of the posterior chamber of the eye, and covering two-thirds of it, lies the retina, a tender reticular membrane consisting of an expansion of the optic nerve. Exactly in the centre of the retina, and consequently opposite to the pupil, there is a small hole, or transparent spot, called the *foramen centrale*, over which the membrane forming the retina does not pass. Between

Fig. 2.



the retina and the sclerotic coat, or white portion of the outer surface, intervenes a delicate membrane called the choroid coat, which, on the side next the retina, is covered with a black pigment. Figure 3 is a

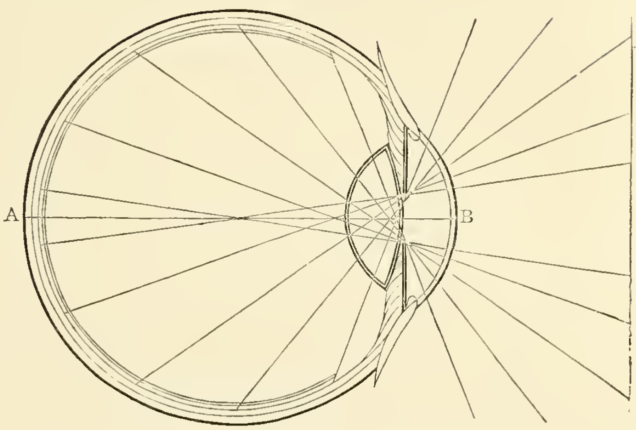
Fig. 3.



vertical section of the eye, on which A is the white or sclerotic coat, B the iris, C the pupil, D the cornea, E the anterior chamber, F the posterior chamber, G the crystalline lens, H the retina, I the choroid coat, and K the *foramen centrale.*

These parts perform the functions of seeing in the following manner:—When rays of light, either natural or artificial, direct or reflex, fall upon opaque objects, they are reflected in all directions from every illuminated part of such objects. But they pass through every transparent body, either in a direct or refracted line, according to the nature of the substance. Therefore such rays as fall upon the white of the eye, or upon the iris, are reflected back again, or partially absorbed; while those that fall within the range of the pupil are transmitted through the aqueous humour, crystalline lens, and vitreous humour, all being transparent media, until they reach the back of the posterior chamber, where they act upon the retina, and are absorbed by the black pigment behind it. This action upon the retina is the depicting of the configuration of any object upon its surface, in the same manner that objects are depicted within a camera obscura with a concave table. The three transparent media have various powers of refraction, by which the rays that enter the pupil converge and are spread upon the retina. From the structure of the human eye, it will be observed that it is limited in its range of perception—that is, that the rays which necessarily traverse each other in passing through the crystalline lens, or one of the other two transparent media, must enter the posterior chamber at a certain angle to reach the retina. And as that membrane covers more than a half of this chamber, the rays, as they approach its extremities, must, from their obliquity, form a gradually less distinct impression. Hence, the dying away of objects around the focal range of sight. The greatest angle at which an object can be clearly and distinctly appreciated by the human eye is 60°—that is, when the rays of light are reflected from every angle in the perimeter of a rectilinear figure, or from every portion of the circumference of a curvilinear one, in such a direction as to form with one another, in passing through the crystalline lens or other media, angles of that number of degrees; which figure will thus occupy about one-third of the posterior surface of the inner chamber of the eye. But we know that objects are seen considerably beyond this range, although in a less distinct manner, and therefore other rays besides those must necessarily reach the retina; and we also know that the eye has the power of concentrating or condensing upon the retina the rays which enter it, in order that minute objects may be more perfectly depicted upon its surface. This is partly performed by the contraction of the pupil. But how rays reach the outer portion of the retina to form the indistinct kind of vision that surrounds the more perfect kind, has not yet been satisfactorily explained. This is, however, of little importance to the present enquiry. The mode in which rays of light may be supposed to enter the eye is shown in figure 4, on which the line A to B is the *optic axis*. It will be observed that these rays traverse each other at three different points upon this axis; but as to which of these points is that upon which the rays really do so, writers on the

Fig. 4.



physiology of the eye differ: however this may be, we know from our own sensations, that the rays must reach the surface of the retina somewhat in the manner here explained, being at the same time affected by the refractive powers of the various transparent media through which they pass. Eyes differ also in regard to the distance at which objects are most distinctly seen, and this difference no doubt arises from variety in the construction of the transparent media, and also from variety in the degree of convexity of the cornea.

In the essay lately published by me* on this subject, I showed that a representative of every variety of rectilinear figure occurred within a diagram produced by the division of the circumference of a circle into twelve parts, and by drawing from each of those parts two straight lines. In investigating the structure of the human eye, I find

* Natural Principles and Analogy of the Harmony of Form: Edinburgh, 1842.

that the same diagram points out the manner of its division into the various parts by which its extraordinary functions are performed.

Fig. 5.

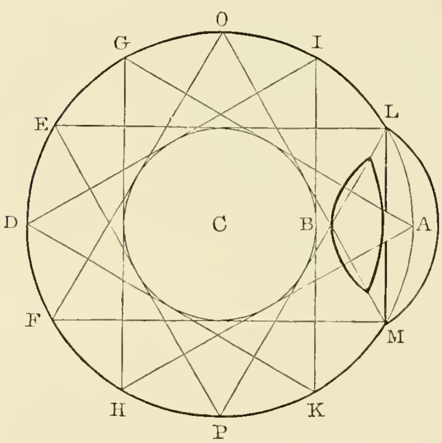


Figure 5 is this diagram, on which A is the centre point of the posterior surface of the crystalline lens; B the centre point of the cornea; C the centre point of the eyeball, and of the anterior surface of the crystalline lens; E D F is that portion of the surface of the retina upon which vision is most perfect; G D H that portion of the surface of the retina which bounds perfect vision; O P the points at which vision almost ceases, and I D K is the extent of the retina; L to M is that portion of the eye which is flat, containing the ciliary process, the iris, and the pupil. The exact relative sizes of the different parts of this organ, are variously stated by writers on its physiology, and the coincidences upon this diagram correspond to those generally given.

Although I have hitherto referred to the effects of forms upon one eye only, in order to be more explicit, these effects are much modified by the rays entering both our eyes simultaneously; hence the mild and pleasing influence of horizontal composition, and the more powerful and grand impression made by that which is vertical. These are the sensible effects of figure upon the organs of vision, and it is only of such that I mean to treat. My observations can therefore have no reference to any geometrical property in figures beyond what can be superficially depicted, as they are reflected upon the retina; for it is well known that we only find out by experience that bodies possess other dimensions than what may be thus appreciated.

The effects of geometrical configuration on the eye are, in the first instance, regulated by the relation they bear to the conformation of that organ itself; hence the soft influence of those of the curved kind, and the acute and more powerful effect of those whose outlines are composed of angles. On the mode of proportioning these elements of form in the combinations of various figures, their effect upon the eye depends—when a proper mode is adopted, geometric beauty is the result, while the adoption of an improper mode results in deformity.

The first principle and most simple element of geometric beauty is proportion, and this quality cannot exist without variety; for there can be no proportion in one simple homogeneous part, nor in the repetition of such a part. The smallest number of parts by which this element is attainable are two, and the greatest number three. The first is the relation that the length bears to the breadth, and the second is the relative quantity of the three kinds of configuration that are produced by the straight, the angled, and the curved line; the various combinations of which elements lead to infinite variety. Those dimensions must relate to one another agreeably to mathematical laws, which are responded to by an inherent principle in the human mind, correlative to them and regulating every effect of external nature upon our senses. This may be called the first principle of taste in regard to figure, and is possessed by mankind in every phase of variety, and when it develops itself in any high degree, it constitutes genius.

This mathematical proportion of parts is that particular element of beauty called harmony, which, in a general sense, means the adaptation of the parts of any thing one to another, or the just proportion of sounds, colours, forms, words, or sentiments.

Poetry, as already observed, owes its beauty, in the first instance, to a geometric principle of proportion, upon which rules are established for its construction; and that, when these rules are followed, it produces a sensible effect of pleasure upon the ear, independently of the meaning conveyed in the words of which the measure is composed. But in music this geometric principle is much more generally understood, and its rules philosophically determined by the science of acoustics.

Agreeably to the experimental enquiries of natural philosophers into this science, it has been determined that the elements of the art of music are, in the first instance, three in number; and these three primary parts are technically called the tonic, the mediant, and the dominant, expressive of their relative effects in musical composition. These sounds or notes are in the general or diatonic scale; the 1st, 3d, and 5th. And there are, therefore, other intermediate notes that act as connecting links to these primaries, and they are called the 2d, 4th, 6th, and 7th degrees of the scale, for the 8th degree is a repetition of the tonic; and it has been established, that agreeably to the mode in which these sounds are arranged, whether in succession or combination, will their effects be harmonious or discordant, and that this depends upon a geometric principle. The art of music consists in a thorough knowledge of the effects that these elementary parts have naturally upon one another; and the object of musical composition is, in the first instance, to make them pleasing to the ear. Thus far the effects of combined sounds upon the ear are merely sensual, and please or displease in so far as the established rules have been attended to or violated in the mode of their combination, and the degree of these effects will be in the ratio of the sensibility of this organ. Musical composition has no doubt a much higher aim, and produces effects of a more exalted kind, but it would be beyond the object of this essay to treat of them.

By the science of chromatics it has been shown that in colouring also there are three primary elements—blue, red, and yellow; and that the complete scale of the colourist has other four secondary or intermediate hues—purple, orange, green, and neutral. The art of painting teaches the proper use of these, the harmony arising from which is in its simplest elementary kind merely sensual, although, like other harmonies, it can be made, in the hands of men of genius, and combined with subjects of an exalted kind, to produce powerful effects upon the mind through the sense by which it is appreciated.

But there has, as yet, been no systematic arrangement, or geometric principles of proportion, applied to form, by which harmony may with certainty be produced; although it has been universally acknowledged that there is a harmony and discord in the modes of combining forms, as certainly as harmony and discord can be produced by various modes of combining either sounds or colours.

In a former essay already alluded to, I gave a detail of the kinds of lines by which forms are produced, and it will only be necessary here to observe, that in geometry they are understood to be three—the straight line, the crooked line, and the curved line. As the crooked line is, however, but a combination of two or more straight lines, it might be argued that there are only two kinds of lines, the straight and the curved, and that they are correlative to silence and noise in sound, and to light and darkness in colour, especially as a single straight line can enclose no figure; while the whole series are produced within an octave of the circle—a single curved line. But such an enquiry could not result in any practical benefit.

A line and a point, mathematically considered, are individually length without breadth, and position without magnitude; but in the arts they are understood to have a physical existence, and are consequently such as may be drawn with a pen, or any similar instrument, and are therefore, in this essay, so considered.

In attempting to establish a system of linear harmony, applicable to form, of an intelligible and practical kind, geometrical figures must be employed; and in order that such a system may correspond to those that regulate other harmonies, three simple elementary homogeneous parts must be shown to exist.

The distinctions of figures do not seem to rest so much upon the number of lines that compose their circumference, or perimeter, as upon other peculiarities in their configuration. Regular curvilinear figures must have either one or two points as a centre, and have no angles; and regular rectilinear figures must be composed of acute angles, right angles, obtuse angles, or an equal mixture of the acute and obtuse kinds. Amongst these geometrical figures there are three which are perfectly simple and homogeneous in the nature of their configuration, and are, in every other respect, quite analogous to the tonic, mediant, and dominant notes of the diatonic scale of the musician, and to the three primaries, blue, red, and yellow, of the colourist. These figures are the circle, the triangle, and the square, in the relative proportions in which they are given in figures 6, 7, 8.

Fig. 6.

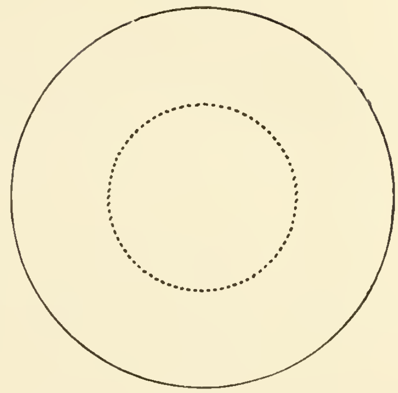


Fig. 7.

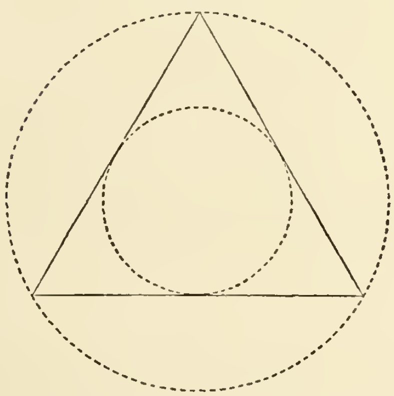
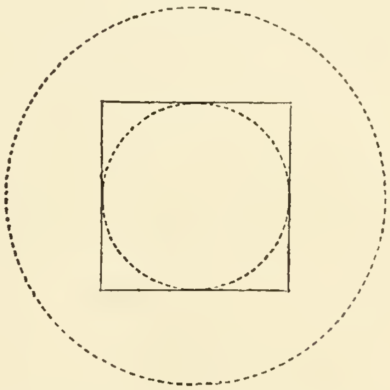


Fig. 8.



I have endeavoured to show in my former essay, that these figures, in such proportions, bear an analogy to the three primary parts of sound in the quantity of their circumference and perimeter. I have also demonstrated, that if two circles be produced from one centre point, having in their circumference the relative proportions of 1 to 2, as in figure 6, those two rectilinear figures, in the proportions in which I have given them, can be placed harmonically between two such circles, as shown by the dotted lines in figures 7 and 8.

The homogeneous simplicity of these figures consists, first, in the circle being the most perfect curve, and composed of one line drawn round one point, from which every portion is equidistant; secondly, in the equilateral triangle being composed of three sides, the smallest number possessed by any rectilinear figure, which sides are equal, and each of which, as well as each of its angles, are equidistant from one point; and thirdly, in the square being composed of four equal sides and four right angles, each side and each angle being also equidistant from one point, and the right angle itself being homogeneous.

Without referring to the analogy of sound, it might be shown that from their configuration, compared to the conformation of the eye, the effects of those particular forms upon that organ entitle them to hold the situation amongst other forms in which I have placed them. The pupil of the eye is circular; hence the rays, or pencils of light, which pass from external objects to the back of the inner chamber, or retina, are most easily transmitted when the object is circular, as already explained. The circle is, therefore, not only geometrically the most simple of the homogeneous forms, but naturally so in reference to the organ by which it is perceived. The square is the next most consonant form to the eye, as its angles, although more in number, are less acute than those of the triangle, and are the exact mean between acuteness and obtusity. The triangle, of the three, is the figure which, from its being composed of acute angles and oblique lines, exercises the most powerful influence on that delicate organ.

In this respect it corresponds to the note E in the diatonic scale in music; for compositions having that note for their key exercise the same relative influence on the ear. Indeed, round and acute are terms as often employed to express qualities of sound, as they are to express the particular configuration of objects presented to the visual organs.

It is well known in chromatics, that the primary colour, blue, exercises a softer influence on the eye than either of the other two, red and yellow; and this no doubt occurs from its being the most allied to darkness or black of the three, and hence associating more intimately with the colour of the retina itself. The colour that stands next to it as a primary in the solar spectrum, is red, which consequently holds the situation that the triangle does in my series of forms; and this colour is well known to affect the eye more forcibly than the yellow, which, in the natural series, is furthest removed from the blue; so that the more acute effect of the triangle upon the eye, although holding a medial situation, is quite in accordance with the analogy of acoustics and chromatics.

The scales by which harmony in sound and colour is produced, as already noticed, have, besides the three primary parts, other four of a secondary kind by which these are connected. So, to complete the scale of forms, it was necessary to adopt figures corresponding to these, as the 2d, 4th, 6th, and 7th notes in music do to the 1st, 3d, and 5th; and as the secondary colours, orange, green, purple, and neutral, do to the primaries, blue, red, and yellow. The figures I now adopt for this purpose are the rectangle, figure 9, the rhombus, figure 10, the hexagon, figure 11, and the dodecagon, figure 12.

Fig. 9.

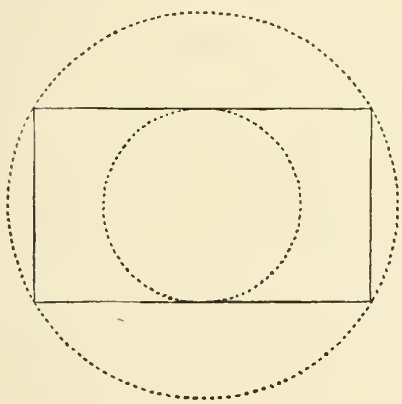


Fig. 10.

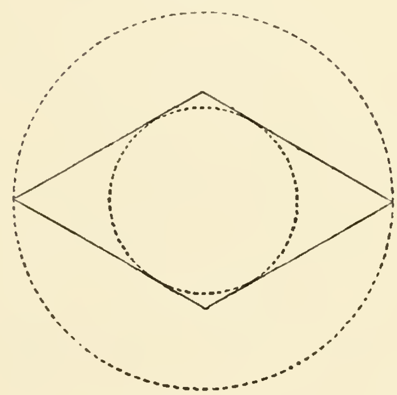


Fig. 11.

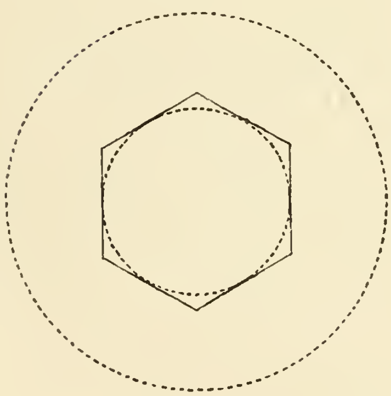
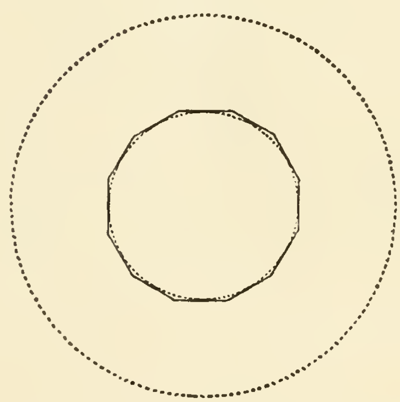


Fig. 12.



These figures, it will be shown, occur naturally from the intersections of two lines produced between two circles of the relative proportions already stated, and form, with the three primaries, a series embracing a representative of every variety of geometric figures, whether right-angled, acutely angled, obtusely angled, or curved.

Forms composed of straight lines are, in geometrical language, called rectilinear figures, and are all necessarily angular, and are named according to the kind and number of angles they contain. These angles are of three kinds, viz. the right angle, the acute angle, and the obtuse angle. The first of these is composed of the two positive directions of the straight line—the horizontal and vertical; and this is the only homogeneous angle, as it admits of no modification in any degree, for whatever direction one of its sides may take, the other must be perpendicular to it. All forms having this angle are called rectangular figures; even a triangle having one of its three angles of this kind, is called a right-angled triangle. The other two—the acute and the obtuse angles—admit of any change in the relative direction of the lines which produce them, without changing their denomination or character. These angles are geometrically regulated by the circle in this way:—Its circumference is divided into 360 equal parts, called degrees, figure 13, which degrees are again divided into 60th parts, called minutes; these, again, into 60 seconds, and these seconds into 60 thirds; and this subdivision may be carried on to any imaginable extent. The circle is divided into two equal parts by a line drawn through its centre, and cutting the circumference at each end, figure 14. This line is in geometry called a diameter, and

Fig. 13.

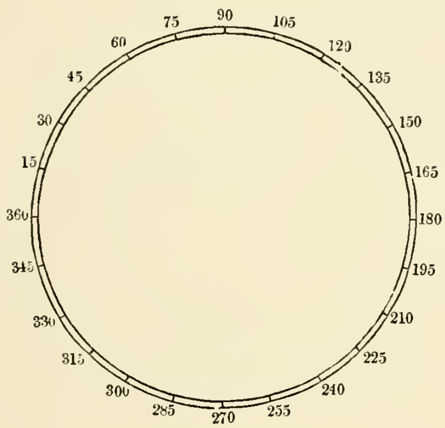
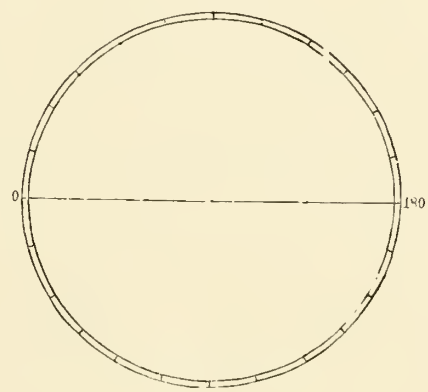


Fig. 14.



when horizontally placed, is the base or groundwork from which all angles arise. The half of the circle is called a semicircle, and the half of the semicircle a quadrant. Any line drawn from the centre of this diameter to the circumference of the semicircle is a radius, and will divide it into two portions called arcs. If it cut the circumference in the centre, these arcs will be equal, and the angles formed with the diameter on each side of the radius will be right angles; and as these arcs contain 90 degrees, right angles are called angles of 90°,

Fig. 15.

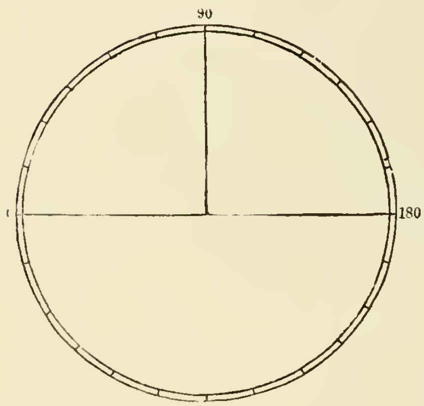


figure 15. But if the radius touch the circumference at any other point than the centre, two different angles are formed; one an acute angle, and the other an obtuse angle, because the one arc contains fewer, and the other more than 90°, as shown in figures 16—17. Upon

Fig. 16.

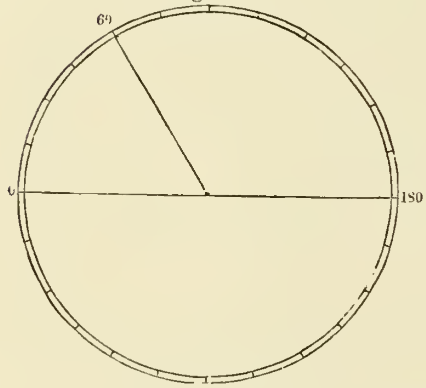
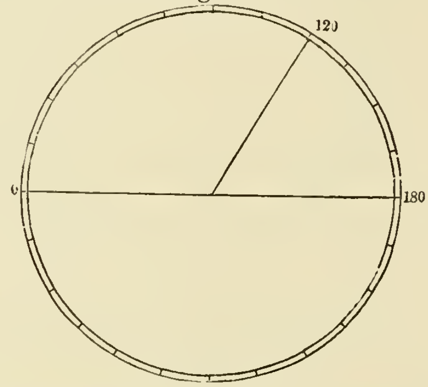


Fig. 17.

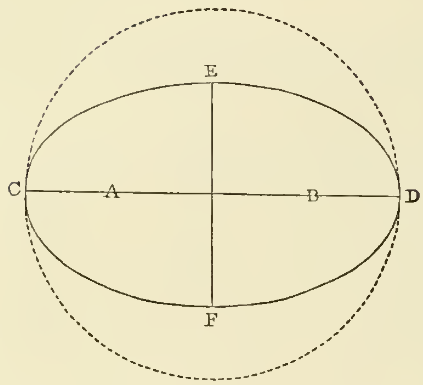


this division of the circumference of the circle depends the character and direction of every straight line, as well as every rectilinear figure. This simple account of the mode in which angles are reckoned, will make the following description of geometric figures more easily understood to those who have not studied geometry.

OF THE CIRCLE AND OTHER CURVILINEAR FIGURES.

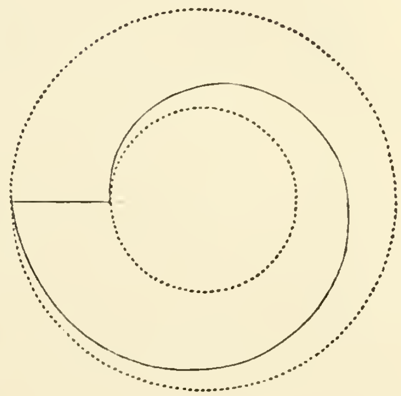
The circle itself, as already shown, is in the parts that compose it the most simple and homogeneous of all forms. Its secondary is the ellipse, also a perfect curve, because it is a line in all its parts equidistant from one or other of two points, and necessarily uniting its beginning and end at the same point. It is heterogeneous, for these points may be placed near or apart; but the figure described around them will still be an ellipse, however much it may resemble a circle from the closeness of its two centre points, or, on the other hand, a straight line from their separation. These points are called its *foci;* and if they remain in the same position while the line which forms the circumference is increased, the ellipses thus formed will continue in appearance to approach the proportions of the circle, but, although produced to infinity, can never form that particular figure. It has consequently two diameters, the longest of which is called the transverse diameter, and the shortest the conjugate diameter, as

Fig. 18.



shown in figure 18, on which A B are the *foci*, C D the transverse diameter, and E F the conjugate diameter. It will be shown in another part, that although this figure can be produced in every variety of proportion, from the straight line to the circle; yet, as it possesses the most simple kind of variety of parts which constitute the first elements of proportion, there are certain rules for its formation which agree mathematically with the principles of geometric beauty, and which entitle the one here given to be termed *the* ellipse, in contradistinction to other varieties of the same figure. In this proportion it is also entitled to hold the place of tonic in the secondary series of geometrical figures, and to produce within its octave the rectilinear figures of that class. The only other curve that can produce a form or figure useful in the arts of design is the spiral: this curve, as well as that producing the circle and ellipse, is a real unmixed curve; and although it can form no figure of itself, is of much importance in the arts of ornamental design in producing the volute. It is a curved line, receding gradually from a focus or centre, figure 19. Its centre may be a point, a circular figure, or an ellipse, and these may be large or small, or its aberration may be in any degree; its direction still forms a spiral line, and the figure it produces when closed by another line, a volute. There are various

Fig. 19.



other curved lines, which, although inapplicable in the arts of design, have been reckoned of much importance in science. They are the cycloid, the parabola, the hyperbola, and others. But all these I look upon as compounds. For instance, the cycloid is the mixture of a straight line and a circle; for during the formation of the circle, by the revolution of a radius around its centre point circularly, that point is traversing a straight line. If the revolution were stopped, and the progress of the radius continued, the ends of the radius would produce two parallel lines. On the other hand, if the progress were stopped, and the revolution allowed to proceed, one end of the radius would describe a circle. But this curve cannot of itself enclose a space or produce a figure.

The parabola, in the same way, is the mixture of an elliptic curve and an angle; or, it may be termed, an ellipse formed upon a definite and an indefinite point, both acting at the same time upon the formation of the curve.

The hyperbola appears to be a mixture of an angle and a circular curve, which, like the ellipse in its production round an elongated centre, will continue to infinity to approach the straight line with which it is associated, as the ellipse has been shown to recede from it.

Every curvilinear figure has a rectilinear figure as its basis, as will afterwards be shown.

OF THE SQUARE, AND OTHER RECTANGULAR FIGURES.

The square is homogeneous in its parts, none of which can be altered without destroying its form. The parts are, as already stated, four straight lines of equal length equidistant from a centre, and uniting at their extremities in four right angles, which are likewise equidistant from the same centre, and being of 90° each, make up the full number 360 contained in the circumference of the circle. When a quadrilateral rectangle has two of its opposite sides longer than the other two, it is called an oblong or right-angled parallelogram; and every rectangle of this kind, from a perfect square to a straight line, is so, whatever may be the proportion between its length and breadth. In the proportion of this figure, therefore, there is the same latitude that exists in regard to the ellipse; and as geometricians have given no rules by which to distinguish *the* parallelogram or oblong from the innumerable series that lie between the square and the straight line, it is of importance to fix some rule for the formation of one whose proportions may be mathematically regulated agreeably to the principles of taste. This I shall attempt in its proper place, and here proceed with a definition of the other primary, and the figures that are allied to it.

OF THE TRIANGLE.

The equilateral triangle is the *proper* triangle; it is, like the other primary figures, homogeneous in its parts, being formed of three straight lines of equal lengths, equidistant from one point or centre, and by their union producing three acute angles, also equidistant from the same point. Like the square, it cannot be altered in any of its parts, without destroying its form and altering its character. Each of its angles are 60°, which together make 180°, being half the number contained in the circumference of the circle.

There are various other triangles, some of which have one right angle and two acute angles; others, one obtuse angle and two acute angles. But they cannot have less than two acute angles, or more than one right or obtuse angle; and, whatever their varieties in other respects may be, their three angles make up 180°. If two straight lines of the same length meet at an angle of 60°, whatever their length may be, a third straight line joining the other two ends will produce an equilateral triangle.

The equilateral triangle has for its secondary the rhombus, which may be termed a perfect mixture of the triangle and square. The ellipse is the secondary to the circle, by having two *foci*, while the circle has only one. The secondary to the square is removed from that figure by having two of its sides shortened, while the number of its angles and the direction of its sides are the same. The rhombus is removed from the equilateral triangle by being two figures of the same kind placed together, which two triangles produce a quadrilateral figure. Two of its opposite angles are 60°, and the other two are 120°. It has, therefore, two acute and two obtuse angles, which, put together, are equal to four right angles. It is the only figure that can occur within the circle, having an equal number of acute and obtuse angles. It may be shortened until its angles be nearly right angles, or it may be lengthened until it approaches the straight line so closely that its figure cannot be distinguished, and it therefore possesses the peculiarities of the other second figures.

OF POLYGONS.

Although this term applies to all figures having more sides than one, yet those whose sides exceed four, are generally denominated polygons; and all regular rectilinear polygons of this kind are consequently obtusely angled. The hexagon has been adopted in the series as the representative of this class, from its being the first regular figure of the kind that occurs from the intersections of the dominant lines, as shall presently be shown. It is the figure that approaches nearest to the configuration of the circle of any rectilinear plane figure that can be joined together in any number by its sides. As polygons increase in the number of their sides, they can scarcely be distinguished from the circle. If two circles be described round one centre point, of the relative proportion in circumference of 1 to 2, and if the outer one be divided into 12 parts, and four lines drawn from each division, these lines will produce between the circle a series of rectilinear figures, three of which will touch the outer circle by their angles, other three will touch the inner circle by their sides, and the centre figure will do both, as appears in figures 20, 21, 22, and 23.

Fig. 20.

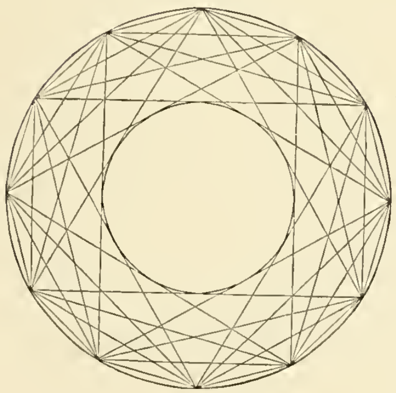


Fig. 21.

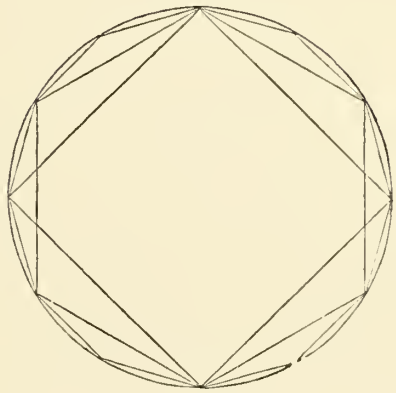


Fig. 22.

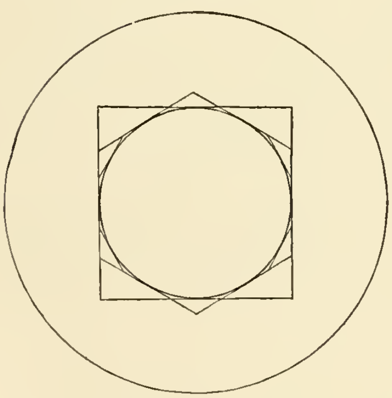
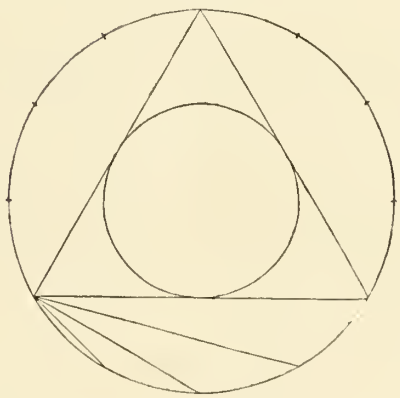


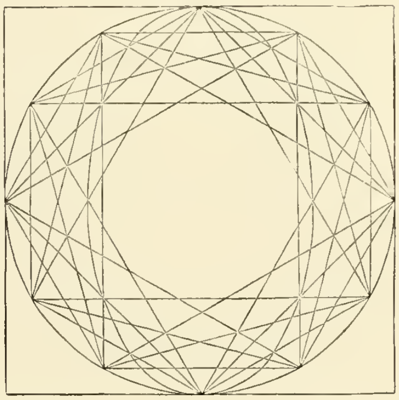
Fig. 23.



Forms must have other distinguishing qualities besides their configuration. We know, that although the diatonic scale of the musician has only seven distinctive characters of notes, this scale may be repeated at least nine times in musical composition between the gravest and the most acute tones appreciable by the human ear; and we also know, that in colours there are many gradations from the deepest and most mellow to the most intensely brilliant of each individual hue. The gradations in both cases are independent of quantity; for a note in music may be long or short, or even strong or weak, without affecting its gravity or acuteness, as a colour may cover a large or a small surface without affecting its peculiar degree of intensity. So it is with form; the three primary figures, with their attendant secondaries, can be produced within the range of ocular perception in an endless variety of combination, and in various degrees of modification in regard to their proportions. To this modification the laws of harmonic ratios shall now be applied.

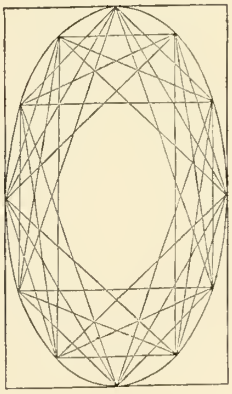
The simple manner in which the rectilinear figures are produced between two circles, has been described above, and that the primary rectangle is formed upon the circumference of the inner circle. It therefore follows, that the curvilinear figure to be adopted as a secondary to the circle, should be an ellipse proportioned to the secondaries of the other two primary figures; that is, it ought to be capable of inscribing the rhombus in such a manner, that the vertices of the angles of that figure should cut its circumference into four equal parts, and of such proportions as may be regularly inscribed by the perimeter of the oblong rectangle, having likewise the relative configuration to the circle shown in figure 24. The other peculiarities of this figure, as well as those of the other secondaries, will be noticed hereafter.

Fig. 24.



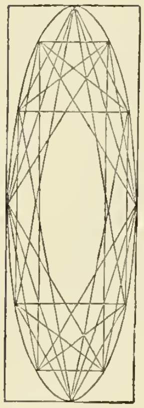
When the circumference of such an ellipse is divided into twelve parts, and lines produced within it as shown in figure 25, it will be found that a secondary series of geometrical figures is formed by their intersections.

Fig. 25.



Amongst these, upon the transversed diameter

Fig. 26.



of the ellipse will be produced an oblong rectangle, which gives the proportions to a third class of figures, as shown in figure 26. The rectangles inscribing those three curvilinear figures, 24, 25, and 26, will be shown to have an harmonious proportion to one another strictly in conformity with the principles of harmonic ratios in form, the development of which is attempted in this essay.

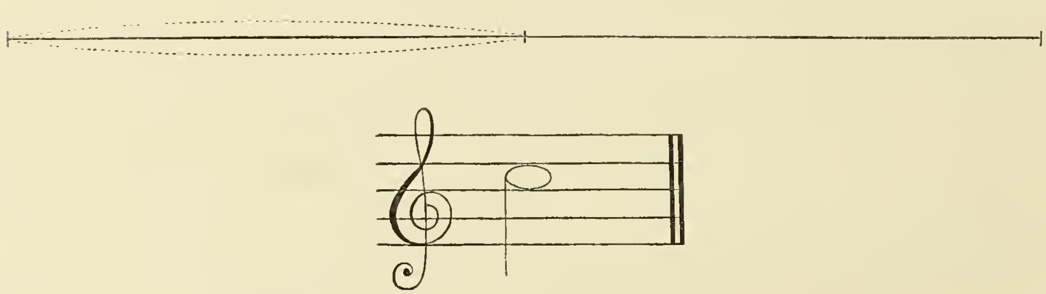
OF THE GEOMETRY OF HARMONY.

We must now turn to the geometry of harmony, in order to find, and more easily to comprehend and systematize, the harmony of geometry. This geometry of harmony, as I have before observed, has formed an interesting branch of natural philosophy, and its laws have been fully demonstrated and established. They are as follow:—The perfect absence of sound is silence, from which point sound commences by the most sonorous or the gravest kind, and ends at the other extreme—the most acute. The one is called the lowest, and the other the highest sound; but as each loses its distinction in silence, these extremes may be said to meet. Natural philosophers have proved that sound is produced by the air being put into a state of undulation, either by the vibration of an elastic body, or of a column of the air itself; and as this undulation does not produce a tone appreciable by the human ear until the vibrations that produce it reach the rapidity of 32 in a second, and that the extreme of appreciable acuteness is almost attained when those vibrations have reached the rapidity of 16,384 in the same period, the range of human perception in regard to sound may be said to be divided into 16,384 parts. This branch of natural philosophy is called acoustics, and by it harmony of sound has been ascertained to depend upon a certain arithmetical division of those vibrations, and that this division bears an inverse ratio to the geometrical division of the body by which the vibrations are produced. For instance, suppose the string of a harp, or any other musical string, to be put into a state of vibration, and if it were of the proper length, thickness, and tension to give 512 vibrations in a second, the sound thus produced would be middle C of the musical scale, or that note which stands intermediately in the range of human perception between the gravest and most acute sounds. Figure 27 represents this string and the note which it produces—the dotted line showing the motion of vibration.

Fig. 27.

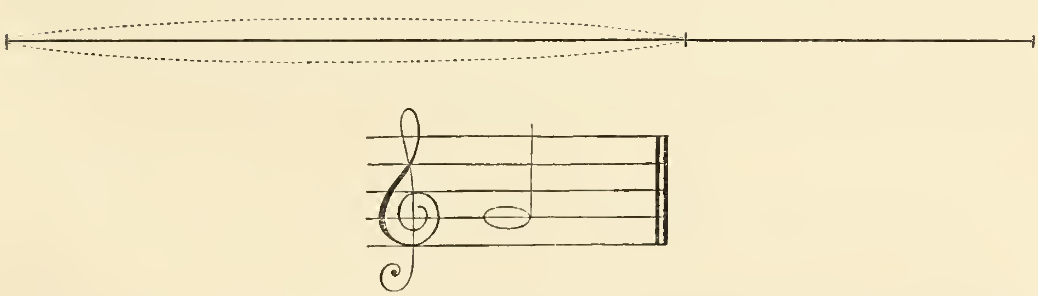
If this string were divided into two parts, and one of those parts rendered quiescent, the rate of vibration would be doubly rapid, and it would consequently give 1024 vibrations in a second;

Fig. 28.



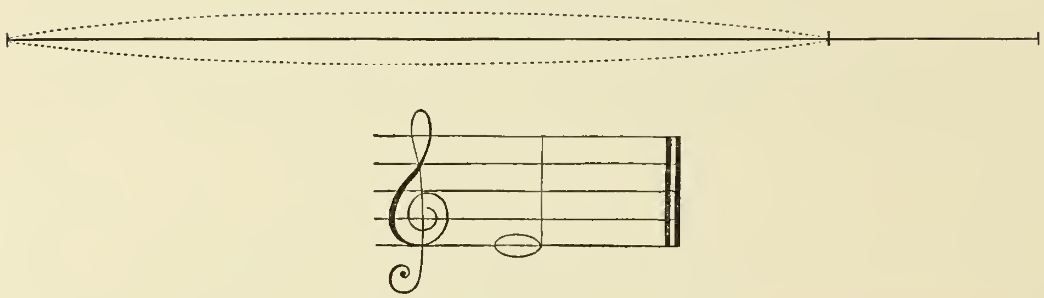
thus producing the next C above it in the scale, which is called an octave to the tonic; because, between it and the original note, the other six, D, E, F, G, A, B, can be produced, being in all eight. This sound is the same note, though doubly acute; and therefore the most consonant sound that can accompany that produced by the whole string or monochord. And it is the case with all sounds within the range of human perception, that, when the number of vibrations in a second are as 2 to 1, the octave or scale is complete. This is the first and most simple division of the string. The second division of the string is by three; and when one of those third parts is rendered quiescent, the relative number of vibrations to those of the original string, or monochord, will be found to be as 3 to 2, and the note thus produced is G, the dominant or fifth degree of the scale, and next most consonant to the tonic.

Fig. 29.



The third division of the string is by 5, and on one of those fifth parts being rendered quiescent, the vibrations will then be found to be in point of rapidity relatively to the tonic as 5 to 4, producing the next consonant note E, the third degree or mediant.

Fig. 30.



If, when the string is divided by three, one-third part instead of two was made to vibrate, the note produced would be the harmonic, called a twelfth, and its vibrations would be to those of the original note as 3 to 1. And if, when the string is divided by five, one instead of four of those divisions was made to vibrate, the note produced would be the harmonic, called a seventeenth, giving vibrations whose rapidity would be relatively to the original note as 5 to 1. These are leading harmonies in the musical scale.*

* A more comprehensive mode of producing the harmonies is as follows:—Taking the same string—one-half gives an octave—one-third part a 12th—one-fourth part a double octave, or 15th—one-fifth part a 17th—one-sixth part a 19th—one-seventh part a 21st, flattened a semitone—and one-eighth part a 22d or treble octave.

It appears, therefore, that harmony in music has a geometrical proportion, and that the division of the vibratory body, and consequently the vibrations it produces, is the most simple, and at the same time the most comprehensive, that could be adopted, viz. by 2, by 3, and by 5; because any other mode of division must be made either by multiples or mixtures of these. Those divisions thus produce the leading tones in all musical composition, and the intermediate notes by which they are connected in the musical scale, are produced by other relative divisions, as shall presently be shown. Now, suppose a string capable of producing, when extended, 32 vibrations in a second of time, which gives the lowest appreciable sound, and that this string or monochord is divided into 16,384 parts, its first division by two will produce an octave above it, giving 64 vibrations in a second, and having 8192 parts; and proceeding to subdivide into octaves the length of string and numbers of vibrations, would stand relatively to one another thus, which is the first and most comprehensive geometry of harmony:—

String.	16384	8192	4096	2048	1024	512	256	128	64	32
Vibrations.	32	64	128	256	512	1024	2048	4096	8192	16384

The more minute kind may be thus detailed within an octave:—

Names of notes in the diatonic scale of music.	Number of vibrations in a second of time.	Relative proportion to key-note.	Number of parts of string.	Relative proportion to monochord.
C, do, or 1st degree	512	1	1024	2
D, re, — 2d —	576	9 to 8	$910\frac{2}{9}$	8 to 9
E, mi, — 3d —	640	5 - 4	$819\frac{1}{5}$	4 - 5
F, fa, — 4th —	$682\frac{2}{3}$	4 - 3	768	3 - 4
G, sol, — 5th —	768	3 - 2	$682\frac{2}{3}$	2 - 3
A, la, — 6th —	$853\frac{1}{3}$	5 - 3	$614\frac{2}{5}$	3 - 5
B, si, — 7th —	960	15 - 8	$546\frac{2}{15}$	8 - 15
C, do, — 8th —	1024	2	512	1

The relative proportions that these notes bear to one another in number of vibrations and parts of monochord are as follows:—

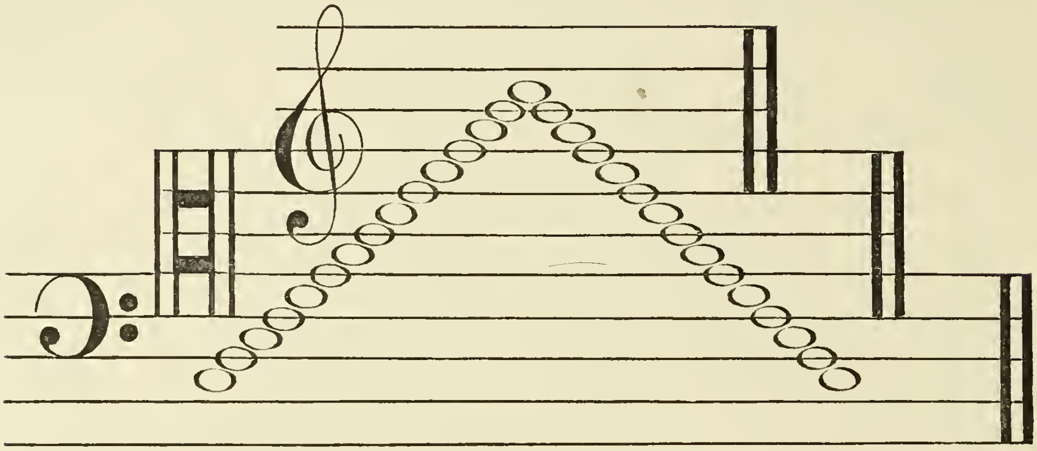
	Relative proportion in number of vibrations.			Relative proportion in parts of monochord.
C to D	8 to 9	called a tone	C to D	9 to 8
D — E	9 — 10	——— tone	D — E	10 — 9
E — F	15 — 16	——— semitone	E — F	16 — 15
F — G	8 — 9	——— tone	F — G	9 — 8
G — A	9 — 10	——— tone	G — A	10 — 9
A — B	8 — 9	——— tone	A — B	9 — 8
B — C	15 — 16	——— semitone	B — C	16 — 15

The following is a Table, showing the number of vibrations in a second of time in each note of the diatonic scale of music, within the range of human appreciation : —

DO	RE	MI	FA	SOL	LA	SI	DO
32	36	40	42⅔	48	53⅓	60	64
64	72	80	85⅓	96	106⅔	120	128
128	144	160	170⅔	192	213⅓	240	256
256	288	320	341⅓	384	426⅔	480	512
512	576	640	682⅔	768	853⅓	960	1024
1024	1152	1280	1365⅓	1536	1706⅔	1920	2048
2048	2304	2560	2730⅔	3072	3413⅓	3840	4096
4096	4608	5120	5461⅓	6144	6826⅔	7680	8192
8192	9216	10240	10922⅔	12288	13653⅓	15360	16384

From the foregoing Table, it will be seen that this scale or series of notes is repeated nine times within the range of human perception. It will, however, be sufficient for our present purpose to take two of those octaves, and arrange them upon the bass, tenor, and treble clefs, which are written thus :—

Fig. 31.



Degrees of Scale.	Vibration.	Relation to Tonic.	Relation to one another.	Relation to Monochord.	String.	Degrees of Scale.
15th cc	1024	4 to 1		1 to 4	512	cc 15th
			15 to 16			
14th b	960	30 ... 8		8 ... 30	$546\frac{2}{15}$	b 14th
			8 ... 9			
13th a	$853\frac{1}{3}$	10 ... 3		3 ... 10	$614\frac{2}{5}$	a 13th
			9 ... 10			
12th g	768	6 ... 2		2 ... 6	$682\frac{2}{3}$	g 12th
			8 ... 9			
11th f	$682\frac{2}{3}$	8 ... 3		3 ... 8	768	f 11th
			15 ... 16			
10th e	640	10 ... 4		4 ... 10	$819\frac{1}{5}$	e 10th
			9 ... 10			
9th d	576	18 ... 8		8 ... 18	$910\frac{2}{9}$	d 9th
			9 ... 8			
8th C	512	2 ... 1		1 ... 2	1024	C 8th
			15 ... 16			
7th B	480	15 ... 8		8 ... 15	$1092\frac{4}{15}$	B 7th
			8 ... 9			
6th A	$426\frac{2}{3}$	5 ... 3		3 ... 5	$1228\frac{4}{5}$	A 6th
			9 ... 10			
5th G	384	3 ... 2		2 ... 3	$1365\frac{1}{3}$	G 5th
			8 ... 9			
4th F	$341\frac{1}{3}$	4 ... 3		3 ... 4	1536	F 4th
			15 ... 16			
3d E	320	5 ... 4		4 ... 5	$1638\frac{2}{5}$	E 3d
			9 ... 10			
2d D	288	9 ... 8		8 ... 9	$1820\frac{4}{9}$	D 2d
			8 ... 9			
1st C	256	1		1	2048	C 1st

The arithmetical relations that these notes bear to the tonic, and to one another, in ascending and descending ; as also, the corresponding geometrical relations in the portions of the monochord by which they are produced, are exhibited in the above Table.

Thus the only kind of harmony, with the philosophy of which we are thoroughly acquainted, and which constitutes the beautiful in sound, owes its excellence to an adherence to certain geometrical rules, which act mechanically upon the ear ; the atmosphere being the medium of communication between that organ and the body which, by its vibrations, produces the sound. This body may be composed of any substance, from a column of the air itself to the hardest stone.

Now it is, as before observed, universally admitted that the eye is as susceptible of what is proportioned or deformed, or, in other words, beautiful or ugly in form, as the ear is to what is harmonious or discordant in sound. Indeed, to such an extent, that the terms which express those sensations are synonymous in both cases. It has already been shown that an analogy exists in the primary and secondary parts employed to produce an effect in either case, and it shall now be attempted to be proved that those parts are equally productive of infinite variety, and equally under the control of certain geometrical principles, in the one case acting mechanically on the eye, as they have been shown in the other to act upon the ear.

OF THE HARMONY OF GEOMETRY.

The circle, as already shown, is geometrically divided into 360 degrees, &c.; and I shall endeavour to prove, that, in the division of those degrees by the harmonic ratios, the principle of geometric beauty or proportion lies.

In the first division by two, which determines the octaves in sound, the diameter of the circle or horizontal line—the base of all geometrical figures—is produced. The second octave gives a radius perpendicular to it, producing the right angle of 90°; and the third, the angle of 45°, which is the diagonal of the primary square. We have thus the first elements of figure; and this division by two gives the first harmonic ratio. Figure 32.

Fig. 32.

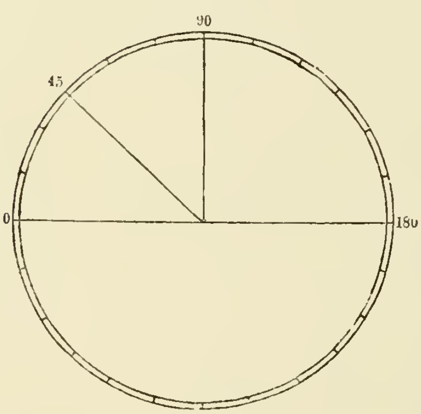
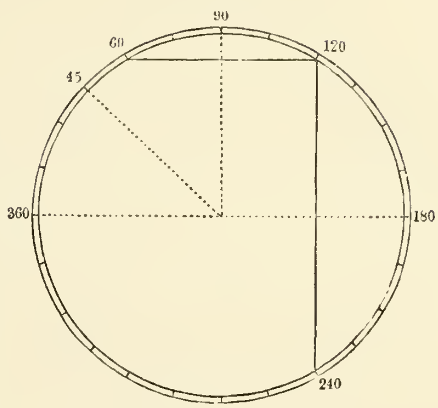


Fig. 33.



The next harmonic division is by three; and when produced upon each of those parts it falls upon the numbers 240°, 120°, and 60°, (Figure 33;) and the lines produced between the first and the second, and between the second and the third of these dominant divisions, are those which, being repeated from each twelfth division of the circle, will produce, by their intersections within it, the diagram which contains the whole series of rectilinear figures, and which has been already so often referred to. The remainder of those divisions will be better understood by what follows regarding rectangles.

Rectangles only differ from one another in their proportion—that is, the ratio that their length bears to their breadth. This proportion is determined by one measurement, which is the diagonal. The difference between the various kinds of angles has been already explained. It has been shown that the angle of 90° is the right angle; and that all angles having more degrees are obtuse, and that those having less are acute. The diagonal, by which the proportion of a rectangle is determined, is a line drawn from the vertex of one of its angles to that which is opposite, and every such diagonal must form two acute angles—the square being the only figure of this kind in which these angles are the same, 45°. This angle is therefore to the angle of 90°, as 1 to 2, and they consequently are in the harmonic ratio of an octave. As no diagonal of a rectangle can be an obtuse angle, all variety of this peculiar line must be found within the quadrant.

The oblong is simply a modification of the square, and this modification is regulated by the number of degrees in the angle of the diagonal, which, when the oblong is placed vertically, must exceed 45°; and, when horizontally placed, must be under that number. If, therefore, a series of these diagonals be produced by an harmonic division of the degrees that occur upon a quadrant—that is, by 2, by 3, and by 5—the rectangles formed upon them must bear an harmonious relation to one another.

The three dominant rectangles already described, were in my former essay proportioned to one another by a different process, but they also agree with this more correct mode—the first being produced by the division of the quadrant by 2, which gives 45°, the diagonal of the primary square, and relates to the right angle as 1 to 2. The second division by 3, gives the vertical diagonal of the first oblong 60°, and relates to the right angle as 2 to 3; and the third division by 5, gives the vertical diagonal of the second oblong 72°, which is in the relative proportion to the right angle of 4 to 5.

Fig. 34.

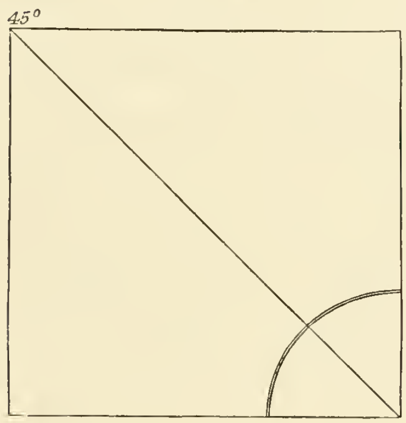


Fig. 35.

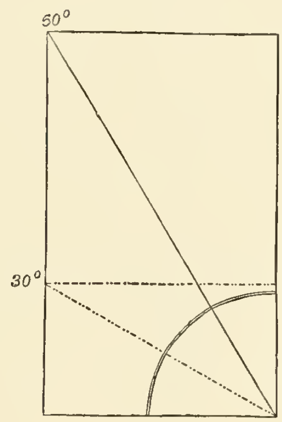
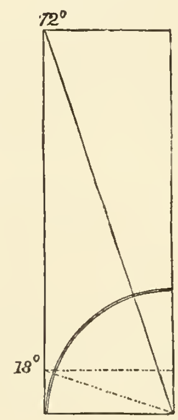


Fig. 36.



The square being homogeneous, has only one proper position, in which its diagonal is always 45°; the other two rectangles being heterogeneous, have two, a vertical and an horizontal. Their vertical diagonals, as shown above, are respectively 60° and 72°. Their horizontal diagonals are therefore 30° and 18°. The first of these latter numbers, 30°, is relatively to its vertical number, 60°, as 1 to 2, and is consequently in the ratio of an octave; and being relatively to 45°

as 2 to 3, it is in the harmonic ratio of a fifth or dominant to that diagonal. Its relation to the tonic, 90°, is that of a musical twelfth, being as 1 to 3.

The second number, 18°, is relatively to its own vertical number, 72°, as a fifteenth or double octave, being as 1 to 4. It is to the tonic, 90°, as a seventeenth, being relatively to it as 1 to 5; and is the third degree to the diagonal 22° 30′, to which it relates as 4 to 5. These and all other harmonic ratios will be found in the following General Scale of geometric degrees, which is divided into nine series or octaves, agreeably to the acoustical division of vibrations:—

First.	Second.	Third.	Fourth.	Fifth.	Sixth.	Seventh.	Eighth.
360°	320°	288°	270°	240°	216°	192°	180°
180	160	144	135	120	108	96	90
90	80	72	67 30′	60	54	48	45
45	40	36	33 45	30	27	24	22 30′
22 30′	20	18	16 52 30″	15	13 30′	12	11 15
11 15	10	9	8 26 15	7 30′	6 45	6	5 37 30″
5 37 30″	5	4 30′	4 13 7 30‴	3 45	3 22 30″	3	2 48 45
2 48 45	2 30′	2 15	2 6 33 45	1 52 30″	1 41 15	1 30′	1 24 22 30‴
1 24 22 30‴	1 15	1 7 30″	1 3 16 52 30⁗	0 56 15	0 50 37 30‴	0 45	0 42 11 15

It has been shown (pages 36, 37, and 38) that the three leading harmonies, agreeably to the established laws of acoustics, are produced by portions of the monochord relating proportionally to one another, in the first instance, or within an octave, as 1 to 2, 2 to 3, and 4 to 5, and are called the 8th, 5th, and 3d degrees of the diatonic scale. But that, when the portions of the monochord are as 1 to 2, 1 to 3, and 1 to 5, the harmonies of an 8th, a 12th, and a 17th, are produced. This is, therefore, precisely the case in regard to the formation of these three rectangles. The angles of their diagonals, in the first instance, relate to the right angle as 1 to 2, 2 to 3, and 4 to 5. But when the horizontal, instead of the vertical diagonal, is employed in the construction of the two latter figures, the three will be found to relate to the right angle as 1 to 2, 1 to 3, and 1 to 5. The harmonic ratios are therefore in this instance quite analogous.

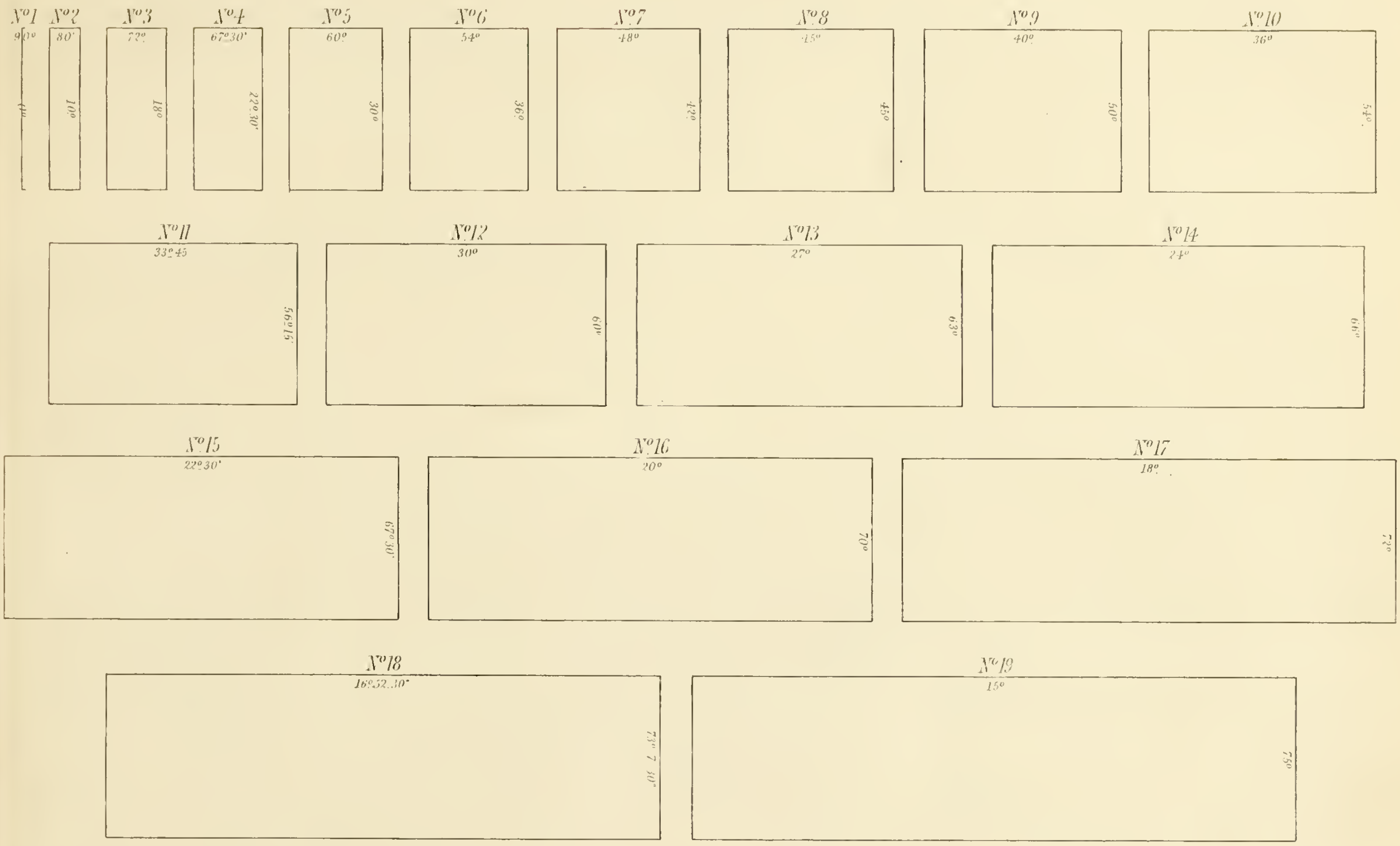
These and the other harmonious relations of the diagonals are apparent in the following comparative scale, in which those from 90° to 11° 15″ are compared with the acoustical and geometrical construction of three octaves of the musical scale. The degrees are numbered from the bottom upwards, in order that they may correspond to the manner in which the notes are arranged upon the clefs, and the degrees in the diagonals are arranged in the same order, that they may correspond to the parts of the monochord.

COMPARATIVE SCALE.

Musical Sounds. Degrees of the Scale.	Ratio of Vibrations to the Tonic.	Number of Vibrations in a Second.	Relations to one another.	Geometric Figures. Number of degrees in diagonals.	Ratio of diagonals to the right angle.	Nos. of rectangles.
22d	8 to 1	1024		11° 15′	1 to 8	22d
			15 to 16			
21st	60 ... 8	960		12	8 ... 60	21st
			8 ... 9			
20th	20 ... 3	853$\frac{1}{3}$		13 30	3 ... 20	20th
			9 ... 10			
19th	12 ... 2	768		15	2 ... 12	19th
			8 ... 9			
18th	16 ... 3	682$\frac{2}{3}$		16 52 30″	3 ... 16	18th
			15 ... 16			
17th	20 ... 4	640		18	4 ... 20	17th
			9 ... 10			
16th	36 ... 8	576		20	8 ... 36	16th
			8 ... 9			
15th	4 ... 1	512		22 30	1 ... 4	15th
			15 ... 16			
14th	30 ... 8	480		24	8 ... 30	14th
			8 ... 9			
13th	10 ... 3	426$\frac{2}{3}$		27	3 ... 10	13th
			9 ... 10			
12th	6 ... 2	384		30	2 ... 6	12th
			8 ... 9			
11th	8 ... 3	341$\frac{1}{3}$		33 45	3 ... 8	11th
			15 ... 16			
10th	10 ... 4	320		36	4 ... 10	10th
			9 ... 10			
9th	18 ... 8	288		40	8 ... 18	9th
			8 ... 9			
8th	2 ... 1	256		45	1 ... 2	8th
			15 ... 16			
7th	15 ... 8	240		48	8 ... 15	7th
			8 ... 9			
6th	5 ... 3	213$\frac{1}{3}$		54	3 ... 5	6th
			9 ... 10			
5th	3 ... 2	192		60	2 ... 3	5th
			8 ... 9			
4th	4 ... 3	170$\frac{2}{3}$		67 30	3 ... 4	4th
			15 ... 16			
3d	5 ... 4	160		72	4 ... 5	3d
			9 ... 10			
2d	9 ... 8	144		80	8 ... 9	2d
			8 ... 9			
1st	1	128		90	1	1st

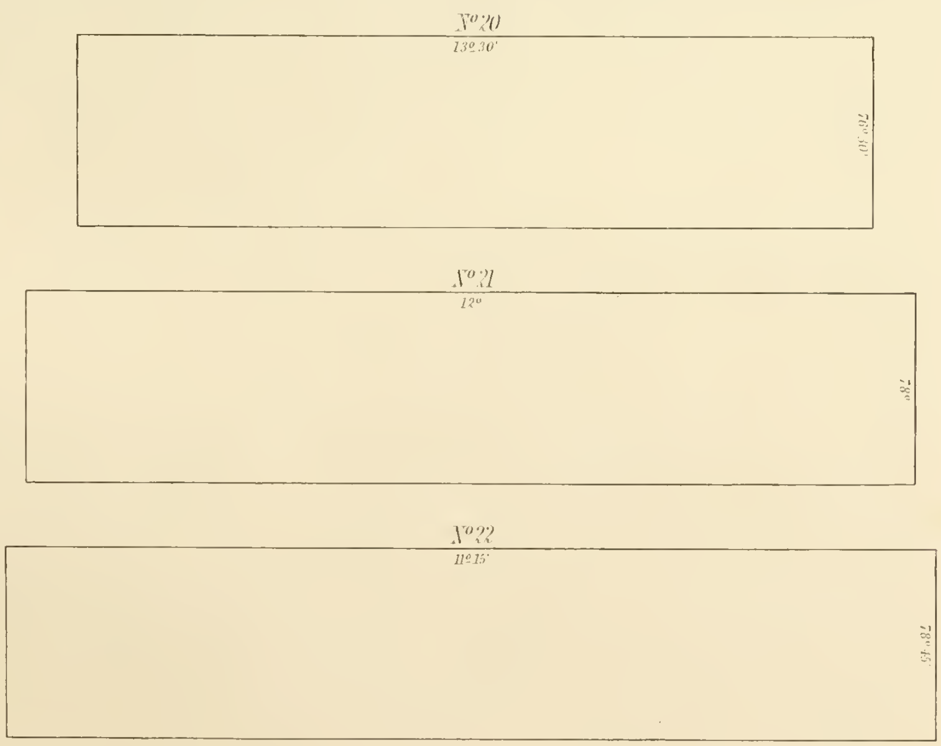
PLATE I.

Rectangles produced by the diagonals in the foregoing comparative scale



Quantities in diagonals as they proceed from the Vertical line

Nº 1	90º	
2	80º	Vertical
3	72º	
4	67º 30'	
5	60º	Rectangles
6	54º	
7	48º	
8	45º	Square
9	40º	
10	36º	
11	33º 45'	
12	30º	Horizontal
13	27º	
14	24º	
15	22º 30'	
16	20º	
17	18º	
18	16º 52' 30"	Rectangles
19	15º	
20	13º 30'	
21	12º	
22	11º 15'	

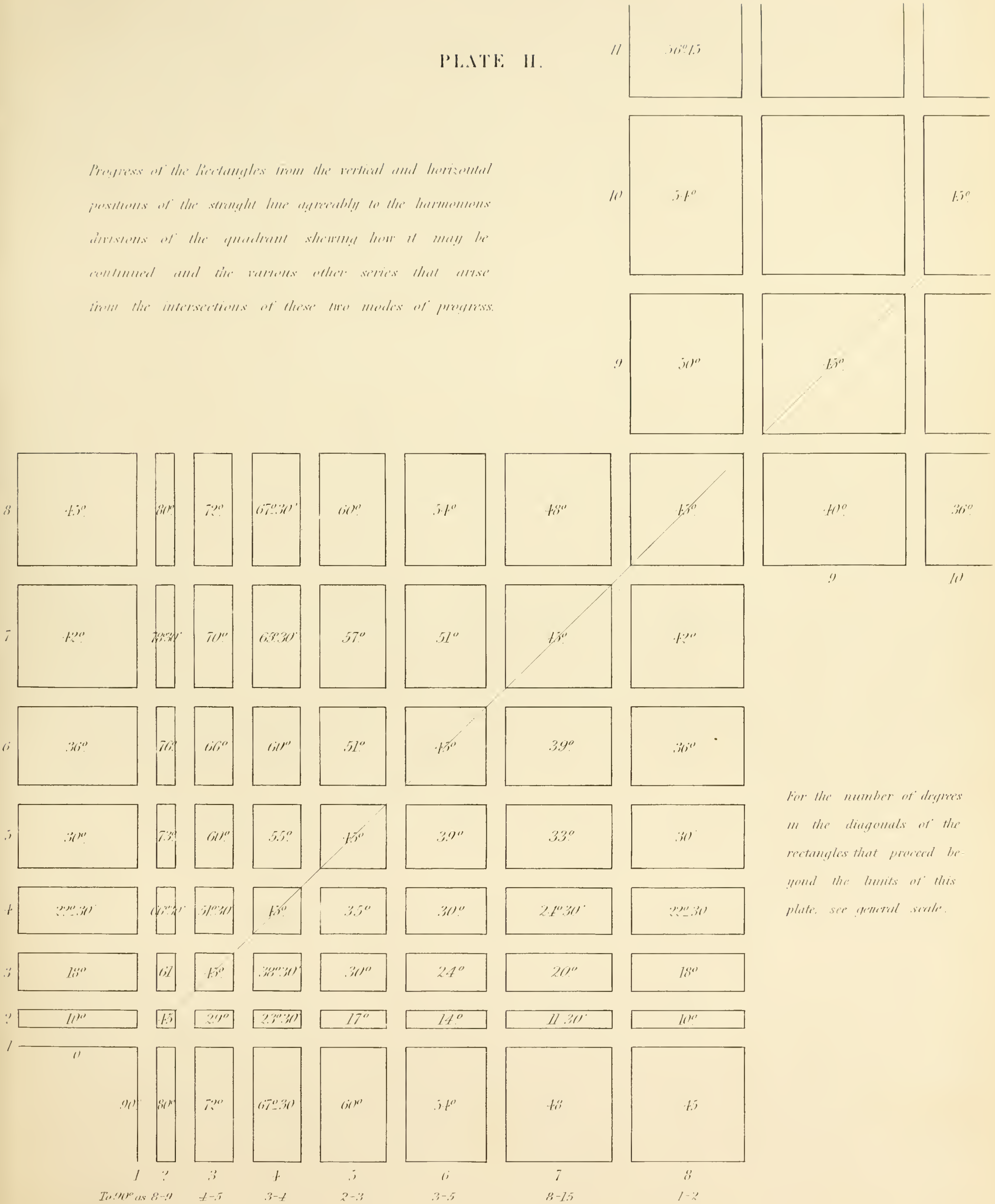


Quantities in diagonals as they proceed from the Horizontal line

Nº 1	0º	
2	10º	Horizontal
3	18º	
4	22º 30'	
5	30º	
6	36º	Rectangles
7	42º	
8	45º	Square
9	50º	
10	54º	
11	56º 15'	
12	60º	Vertical
13	63º	
14	66º	
15	67º 30'	
16	70º	
17	72º	
18	73º 7' 30"	Rectangles
19	75º	
20	76º 30'	
21	78º	
22	78º 45'	

PLATE II.

Progress of the Rectangles from the vertical and horizontal positions of the straight line agreeably to the harmonious divisions of the quadrant shewing how it may be continued and the various other series that arise from the intersections of these two modes of progress.



For the number of degrees in the diagonals of the rectangles that proceed beyond the limits of this plate, see general scale.

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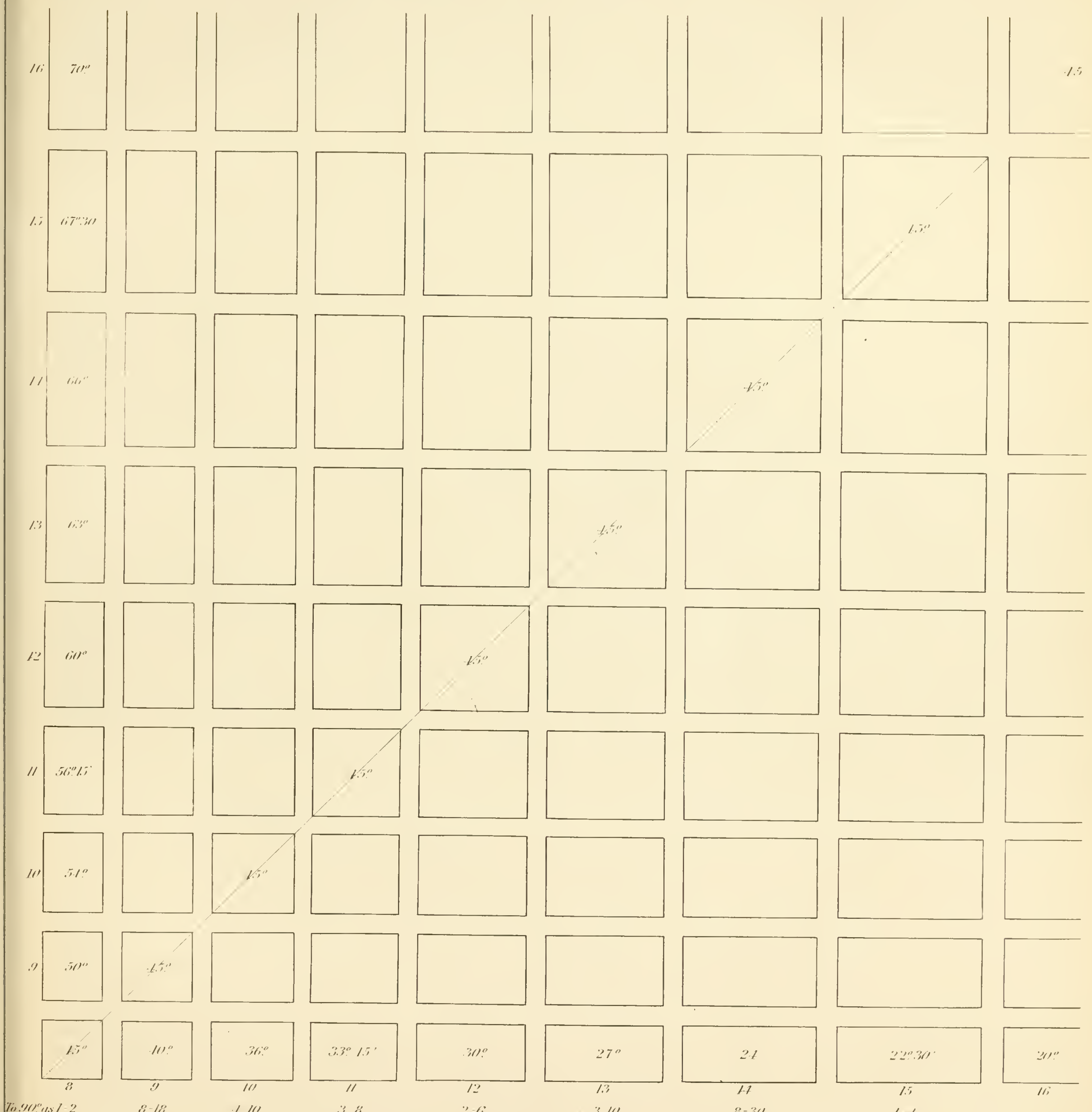
PLATE III.

Progress of the Rectangles from the homogeneous square, being a continuation of the series commenced in Plate II half size



For number of degrees in the diagonals of other rectangles see general scale.

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In Plate I., I give the rectangles produced upon the diagonals in the comparative scale, with their vertical and horizontal quantities marked upon them, by which it will be seen, that those figures, in their natural progress from 90°, increase in quantity of superficies inversely to the number of degrees in the diagonal upon which they are formed, in the same manner that the notes in music increase in the numbers of their vibrations inversely to the parts of the monochord producing them, although the ratios differ.

In Plate II., I have arranged them in such a manner that they proceed from the horizontal and vertical lines, forming with one another various intermediate series, which mode of progression may be continued to any imaginable extent. In this series, the right angle is the tonic, or key.

In Plate III., which is a continuation of Plate II., 45° may be considered the key of the diagonals, and consequently the square is the key of the rectangles. It will be observed, that a different mode of progression is adopted from that of the first series, which may also be continued to the most minute division of the quadrant; but these two series are sufficient to show the harmonious progression of this ruling figure, and the endless, though systematic, variety to which the harmonious division of the geometric quadrant leads.

In my former work on this subject, I divided the musical scale into twelve equal parts or semitones, giving two of those to each full tone, and upon this basis I endeavoured to found an analogy. But this mode of division of the musical intervals was not correct, and the work was scarcely published before I detected this apparent mistake. I say apparent, because it appears that this mode of division is nearly correct in a geometrical point of view, and arises out of the true mode of division in the following manner:—Let a quadrant be divided into two parts, and let one of those parts be subdivided into the correct divisions of the monochord already exemplified; let a line be drawn above the quadrant, and parallel to its base, and the diagonals which cut the arc of the quadrant at the proper divisions, will, when they meet this line, divide it very nearly into the equal tones and semitones adopted in my former Essay.

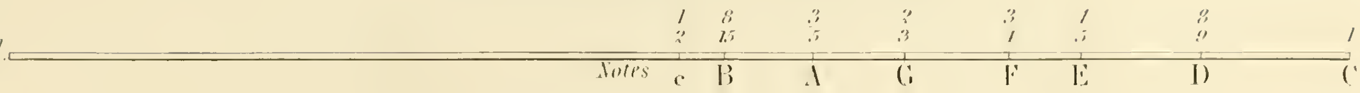
Thus are the diagonals, in the degrees of their obliquity, equal to the correct musical intervals, as also to the lengths of string by which

PLATE IV.

Acoustical and Geometric harmony compared.

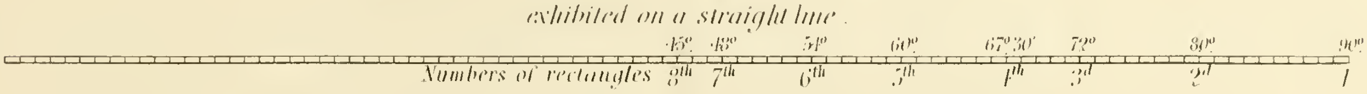
Relative lengths of musical strings required to produce the diatonic scale of notes.

Figure 1.



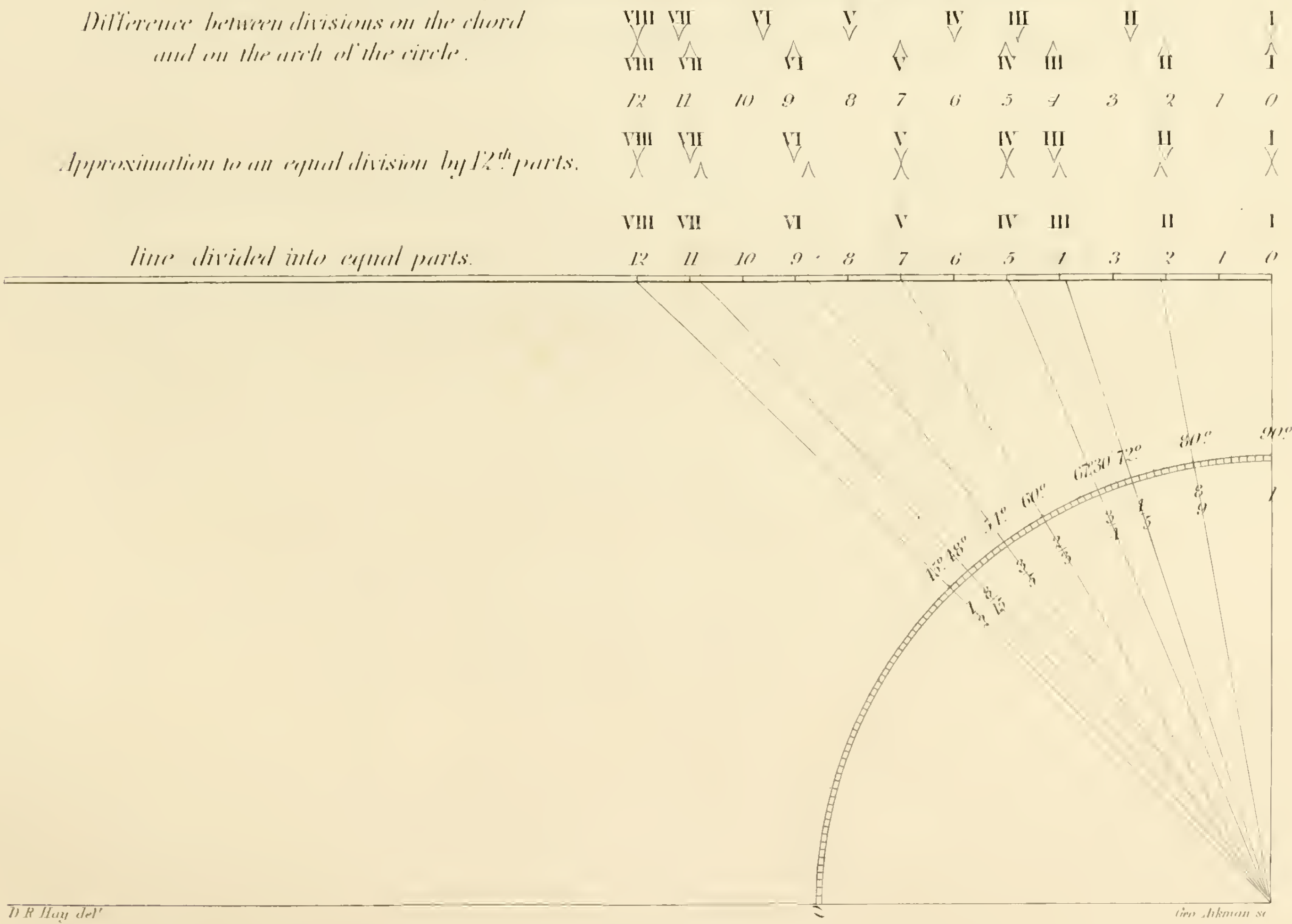
Relative numbers of geometric degrees required in diagonals to produce the first series of rectangles, exhibited on a straight line.

Figure 2.



Geometrical relations of the diagonals which produce the first series of rectangles to the Harmonic division of the monochord by which the diatonic series of musical notes is produced.

Figure 3.



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the intervals are produced; while the rectangles formed upon those diagonals, which differ from one another only in one dimension, have that difference nearly in equal sixth and twelfth parts. Therefore, although my former mode of division was upon a wrong system, the results were correct in regard to the harmonic ratios of the rectangles, upon which class of figures, any attempt to systematize the harmony of form must depend.*—See Plate IV.

As, agreeably to the laws of acoustics, it was shown that the various divisions of the monochord produced an harmonious arrangement of

* In a review of my former work, which appeared in the *Athenæum*, June 10th and 24th, alike distinguished for its candour and ability, the writer points out this error in my mode of dividing the musical scale, which certainly affects seriously the analogy I attempted to establish in that very imperfect production. And he is quite right in supposing that I was misled by the popular mode of speech which represents the octave as composed of five equal intervals and two half intervals, making up six full tones. His observations upon this subject are so clear and decisive, that, an earlier acquaintance with them would have enabled me greatly to improve the preceding part of this Essay; for the existence of a perfect analogy between the perimeters of my seven figures and the musical intervals is not at all essential to my present theory, and I might therefore have dispensed with further allusion to it. But it is not so with the harmonic ratios adopted in this Essay for regulating diagonals, for they appear to me essential in reducing any theory of this kind to system, without which it can be of little practical use.—*26th June* 1843.

sound which may be either employed in combination or succession, according to certain rules; so has the harmonious division of the circle in the first instance produced the three kinds of right lines—the horizontal, the vertical, and the oblique; in the second, the leading figure in every variety of rectilinear form; and, in the third, a proper series of diagonals, by which every modification of those forms may be regulated. Because the rectangles are, in every arrangement of form, the dominant figures, in so far as they rule the proportions of all others, in the same way that the curves of the circle and ellipse have been shown to rule the configuration of all rectilinear forms.

The circle and the square seem to have a reciprocal effect upon one another in regard to this harmonious mode of division; for if the perimeter of the latter be divided into sixteen equal parts, a circle drawn within it tangential to its sides, will of course be divided at its points of contact into four equal divisions or quadrants. But if parallel lines be drawn across the area of the square from the other divisions of its sides, these quadrants of the circle will each be divided into three equal parts—Figure 37; and if from the points at which these parallel lines cut the periphery of the circle, other straight lines are drawn to the points of contact, the diagram, containing a complete series of the rectilinear figures, will again be produced. Figure 38.

Fig. 37.

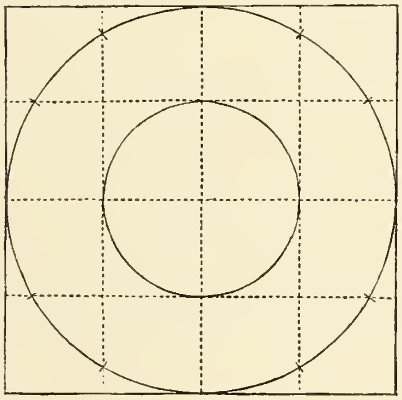
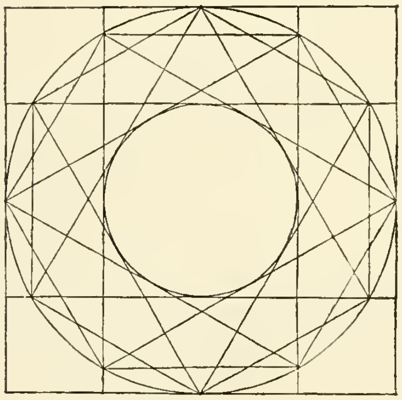


Fig. 38.



These lines form right angles with the side of the exterior square, angles of 30° with any radius that meets them from the centre of the circle, and angles of 60° with the lines drawn to the point of contact, and they inscribe the inner circle by a square half the perimeter of the exterior one.

If the four sides of any rectangle be divided in this manner—that is, each into 4—the ellipse which it inscribes will be harmonically divided into four parts by the points of contact, each of which will be subdivided by parallel lines passing through the area of the oblong into three proportionate divisions; and by uniting the points of contact with the points of intersection, a series of rectilinear figures will be produced, inscribed by the ellipse, and leaving an area in their centre capable of containing another ellipse of precisely the same proportions and half the circumference of the first. This same process may be performed within every rectangle in the series; and the proper angles, obtuse and acute, as well as the proper curve belonging to any rectangle, either vertically or horizontally, thus accurately determined, as shall afterwards be shown.

As within a semicircle, every acute angle formed between a radius and its base, from 0° to 90°, forms on the other side an obtuse angle, the number of degrees in which, added to the acute angle, is always 180°; so within a quadrant every diagonal, from 0° towards 45°, will give the proportions to a horizonal rectangle, and will have on the other side a corresponding diagonal, the number of degrees in which, added to the first, will always be 90°. This diagonal will give the proportions to a vertical rectangle, having a certain harmonious relation to that formed upon the horizontal diagonal, and conversely. These relations are worthy of particular attention, and occur in the first series of rectangles as follows :—

Vertical.		Horizontal.	
I. 90°	...	0°	Being a Tonic.
II. 80°	...	10°	——— 22d, or Treble Octave.
III. 72°	...	18°	——— 15th, or Double Octave.
IV. 67° 30′	...	22° 30′	——— 12th, or Compound Fifth.
V. 60°	...	30°	——— an 8th, or Single Octave.
VI. 54°	...	36°	——— a 5th, or Dominant.
*VII. 50°	...	40°	——— 3d, or Mediant.
VIII. 45°	...	45°	——— an Octave to the Tonic.

The ratios of those numbers are to one another as follows :—

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
0	8 to 1	4 to 1	3 to 1	2 to 1	3 to 2	5 to 4	1 to 1

* The seventh diagonal requires to be lengthened to 50°, in order to produce an harmonic ratio with 40°, which is the corresponding number on the opposite side of 45°.

It has been observed, in the Introduction, that forms and figures as used in the arts require one or other of two qualities to render them pleasing: the first of which is the imitation of natural objects, and the second is harmony, produced by the proportion and arrangement of the elements of abstract form. The one is like a simple description in plain language, while the other is the geometric poetry of the graphic art. When forms are combined merely to produce a pleasing effect, as in ornamental design, or even in architecture, where the subjects are not in imitation of any natural objects, such compositions may be assimilated to those of melody or harmony in music, independently of the analogy of their elements. It has, however, been shown, that the elements of abstract form are in many respects similar to those of abstract sound; and I shall now endeavour to show how those elements may be combined in harmony, and how an endless variety of harmonious combinations may be produced. And, in doing so, I shall endeavour to render the plates as explicit as possible to the general reader, and thereby avoid the necessity of any long explanations.

Plates V., VI., VII., and VIII., are combinations of this kind, and show, that if a quadrant be placed upon any diameter of a circle, and lines drawn through any of the harmonic divisions until they reach the circumference of the circle, and another line drawn from this perpendicular to the diameter or base until it again meet the circumference, the repetition of these two lines from every similar division of the circumference will produce an harmonious arrangement.

If the diagonal, or oblique line, be drawn through the quadrant at 45°, an arc of 90° will be cut off the circle, and the perpendicular to the base that leaves at the point of contact will divide the circumference into two equal parts of 180°; and the repetition of the diagonal at each arc of 90°, will produce a square of exactly half the superficial contents of that which inscribes the circle, corresponding in this respect to the harmonic ratio of a musical octave. Plate V., Figure 1.

If the diagonal be 60°, the vertical, or perpendicular to the base, will cut an arc of 120° off the circle; the repetition of which perpendicular, along with the diagonal, will produce the equilateral triangle, the proper parallelogram, the proper rhombus, and the hexagon. Plate V., Figure 3. Figure 4 shows the harmonious combinations of those lines.

If the diagonal be 72°, the vertical, or perpendicular to the base, will cut an arc of 72° off the circle; the repetition of which perpendicular, along with the diagonal, will produce the pentagon and decagon. Plate VII., Figure 1.

The diagonal lines from the quadrant form the perimeter of the rectilinear figures within the circle; while the diameter of the circle, forming the base of the quadrant, gives the diagonal; indeed, those diagrams are simply circular arrangements of the harmonious rectangles.

These are the three primary divisions, viz. by 2, by 3, and by 5; and the others being merely multiples, or compounds of these, further description than that given upon the plates is unnecessary. It will be observed, however, that when the division of the circumference of the circle is uniform, the intersections form various concentric

PLATE V.

Diagrams of Harmonic angles.

Figure 1.

Division of the quadrant by Two producing the angle of 45°.

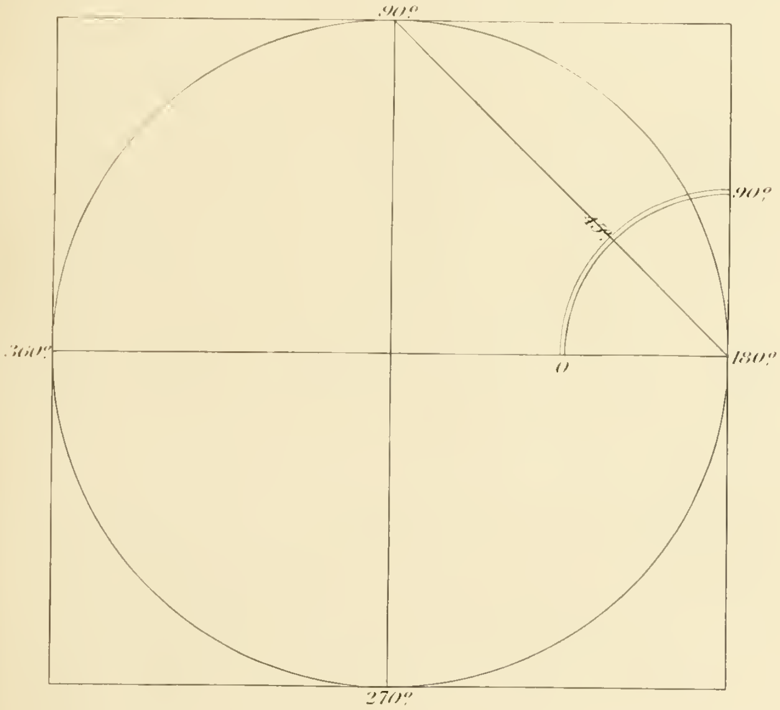


Figure 2.

Diagram produced by the above angle.

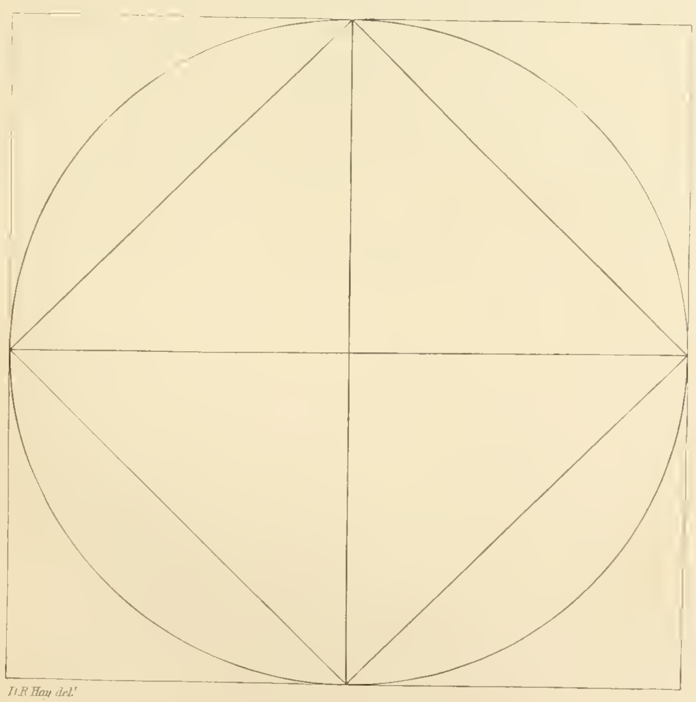


Figure 3.

Division of the quadrant by Three producing the Harmonic angles 30° & 60°.

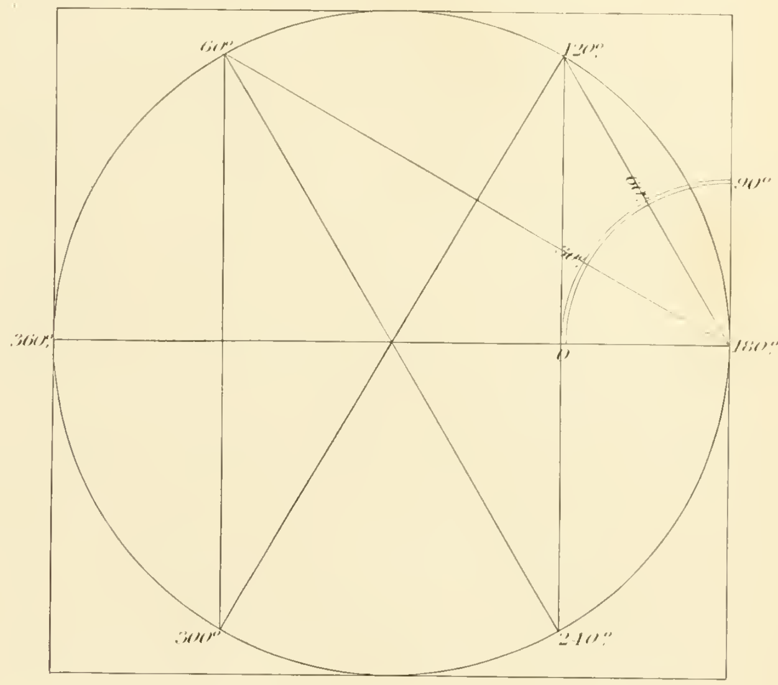


Figure 4.

Diagram produced by the above angles.

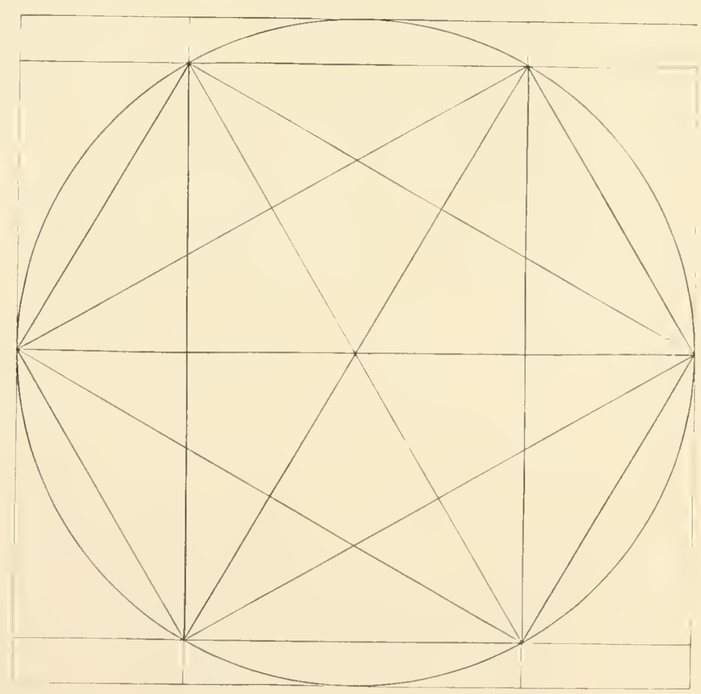


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PLATE VI.

Harmonic Diagrams

Figure 1.
Division of the quadrant by Four producing the Harmonic angles.
22°30', 45° 67°30'.

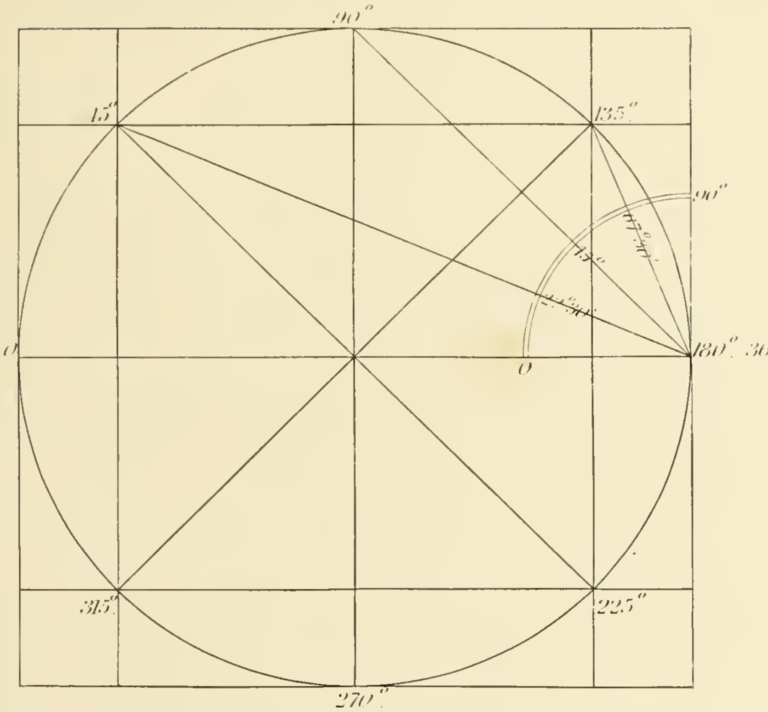

Figure 2.
Diagram produced by the above angles.

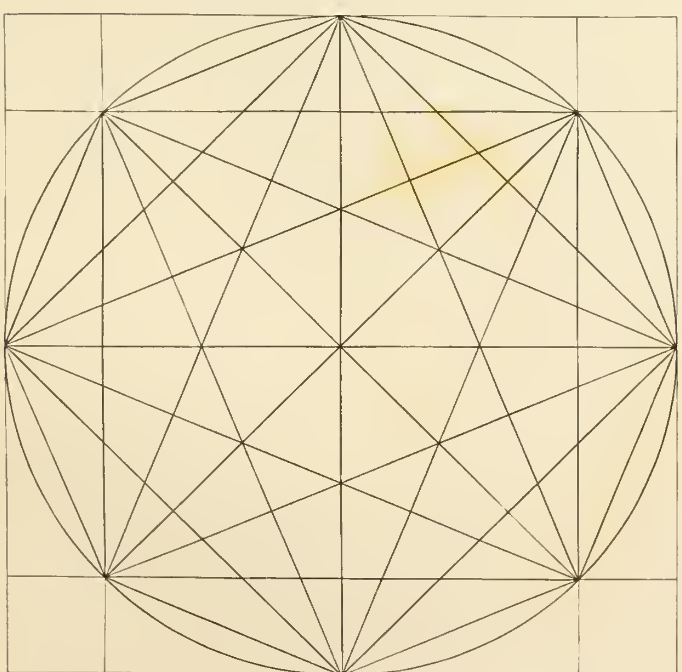

Figure 3
Division of the quadrant by Six producing the Harmonic angles
15°, 30°, 45°, 60°, 75°.

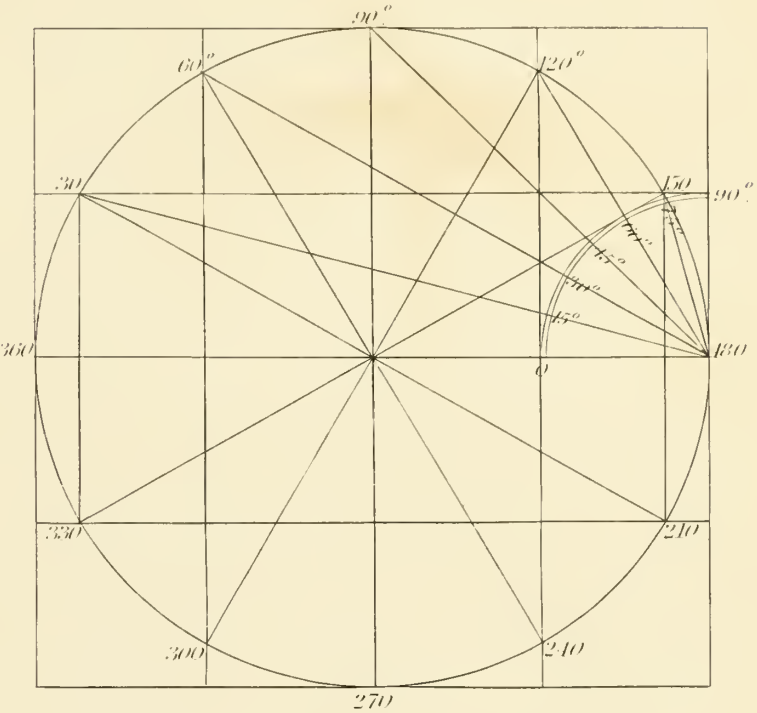

Figure 4.
Diagram produced by the above angles.

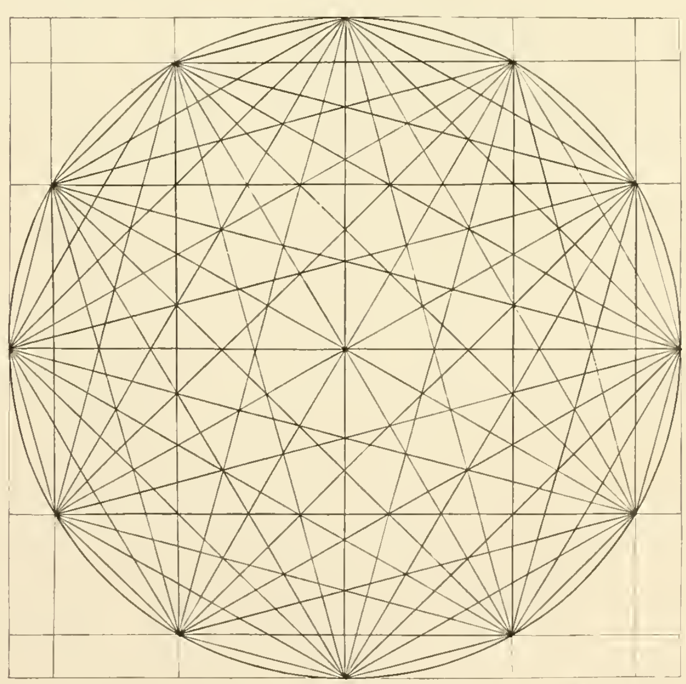

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PLATE VII.

Harmonic Diagrams.

Figure 1.
Division of the quadrant by Five producing the Harmonic angles.
18°, 36°, 54°, 72°.

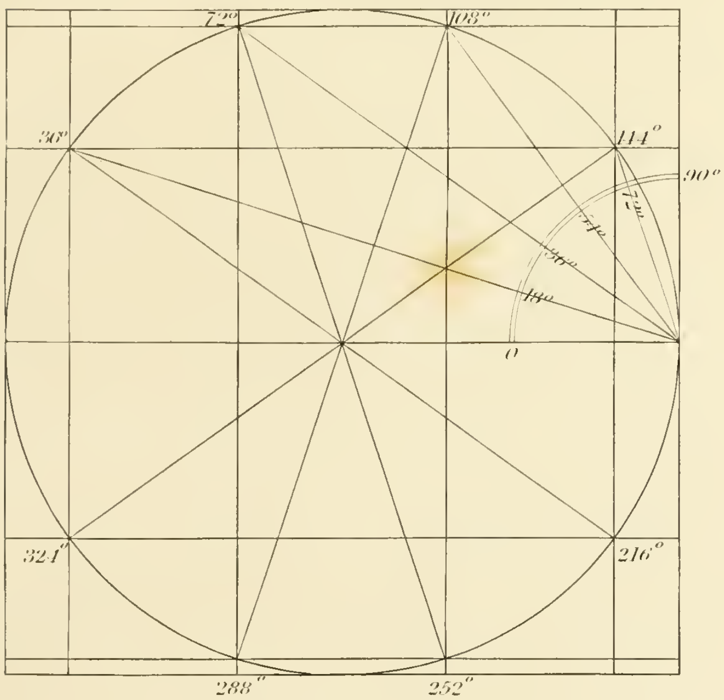

Figure 3.
Division of the quadrant by Ten producing the Harmonic angles.
9°, 18°, 27°, 36°, 45°, 54°, 63°, 72°, 81°.

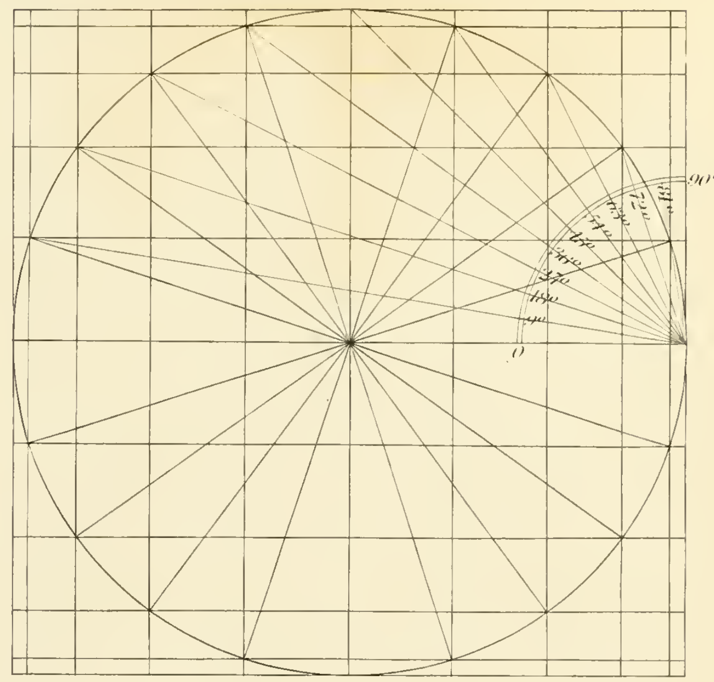

Figure 2.
Diagram produced by the above angles.

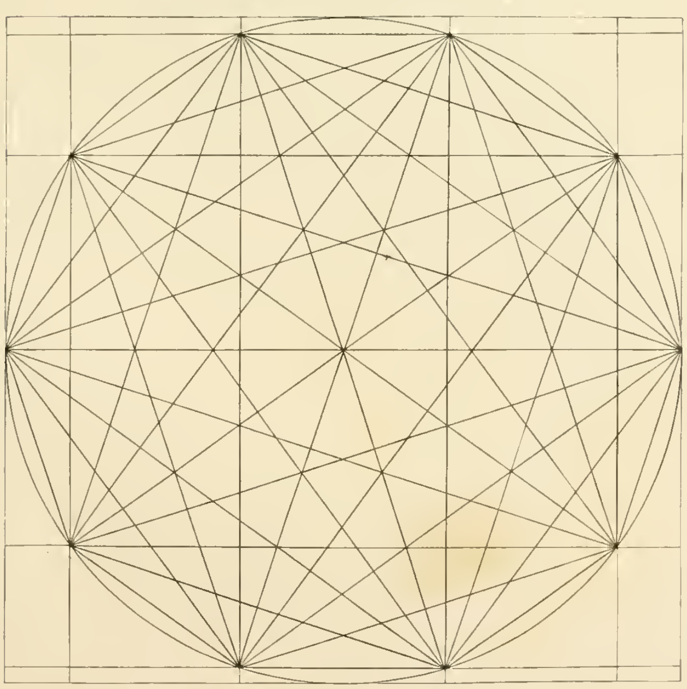

Figure 4.
Diagram produced by the above angles.

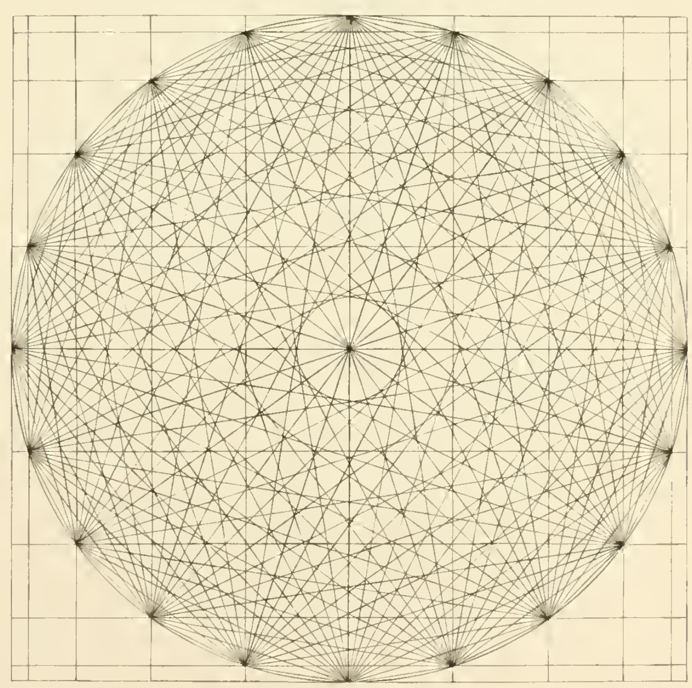

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PLATE VIII.

Harmonic Diagrams.

Figure 1.

Division of the quadrant by Twelve producing the Harmonic angles.
7°30', 15°, 22°30', 30°, 37°30', and 45°.

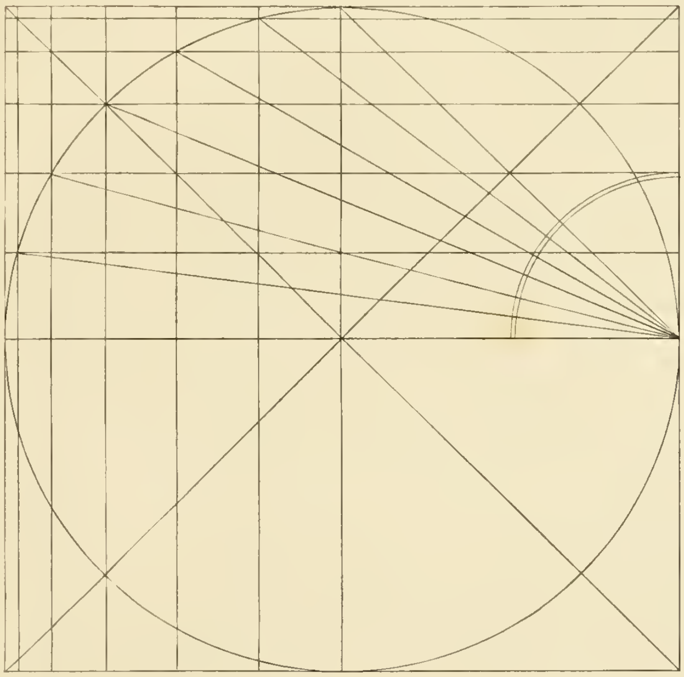

Figure 2.

Diagram produced by the above angles.

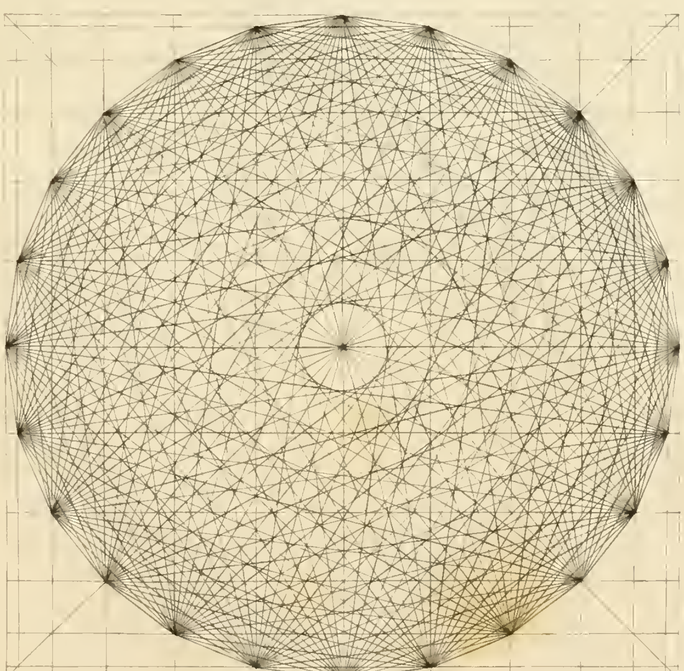

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Figure 3.

Mixture of the Three primary Divisions by 2, 3 and 5 producing
the Harmonic angles 45°, 36°, 30°, 15°, 9° and 22°30'

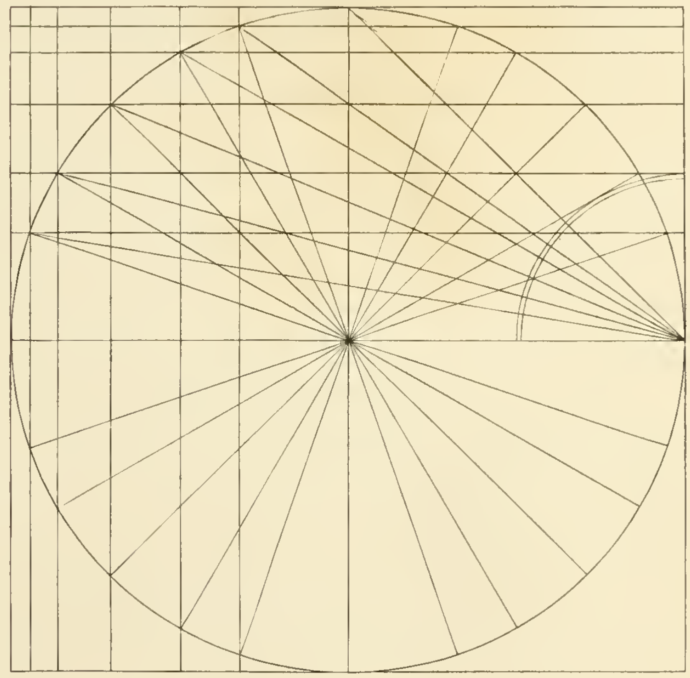
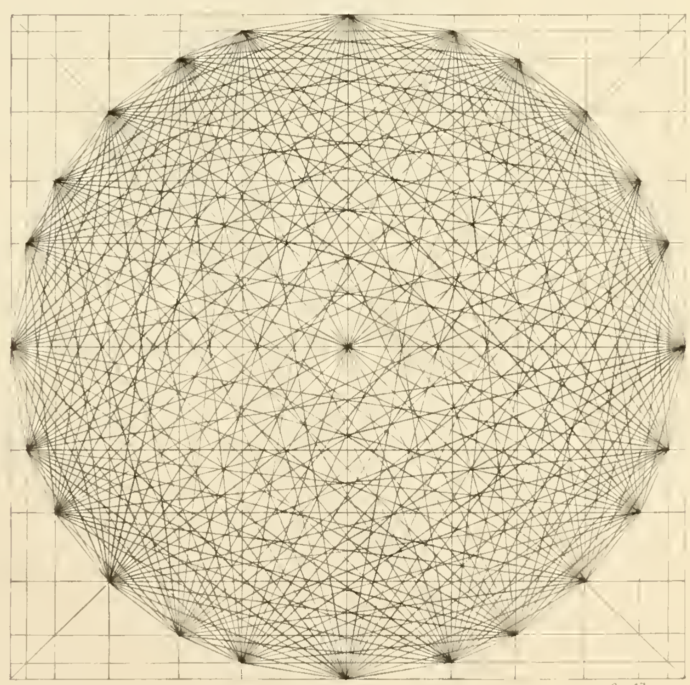


Figure 4.

Diagram produced by the above angles.



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polygons, which approach the figure of the circle so nearly, that they at first sight deceive the eye. As an instance, Figures 2 and 4, Plate VIII., have not a single curved line in their composition. But when the division is irregular, by including the third diagonal of 18°, a different effect is produced, in which no particular figure is distinguishable; and the apparent curves, instead of being circular, are hyperbolic.—Plate VIII., Figure 4. Examples of this kind of harmony might be multiplied to any extent.

Such combinations of lines may be termed *capriccios* in the harmony of form, which, although they display every variety of figure harmoniously arranged, have no subject, and are consequently mere exemplifications of dexterity. Yet are they suggestive of subjects, especially in mere ornamental design, and I shall, therefore, add other four compositions—or more properly combinations—of this kind. They are of a totally opposite character, being angular arrangements of the homogeneous curve, while the former may be termed circular arrangements of the homogeneous angle—the diameters of the circle, or diagonals of the rectangles being superfluous. Plates IX., X., XI., XII.

Such combinations may be multiplied to any extent, and the result never be otherwise than pleasing while the harmonic ratios are attended to. It will be observed in those diagrams, that, as the straight line assumed the appearance of the curve in the circular mode of combination, so does the curve assume the appearance of the straight line in those combinations that are angular.*

Having thus endeavoured to develop my theory, and give a few results of the most simple combinations of the straight and curved lines, agreeably to its rules I shall now endeavour to explain the harmonic ratios of numbers, and show some of the peculiarities of the seven figures which form the basis of my theory.

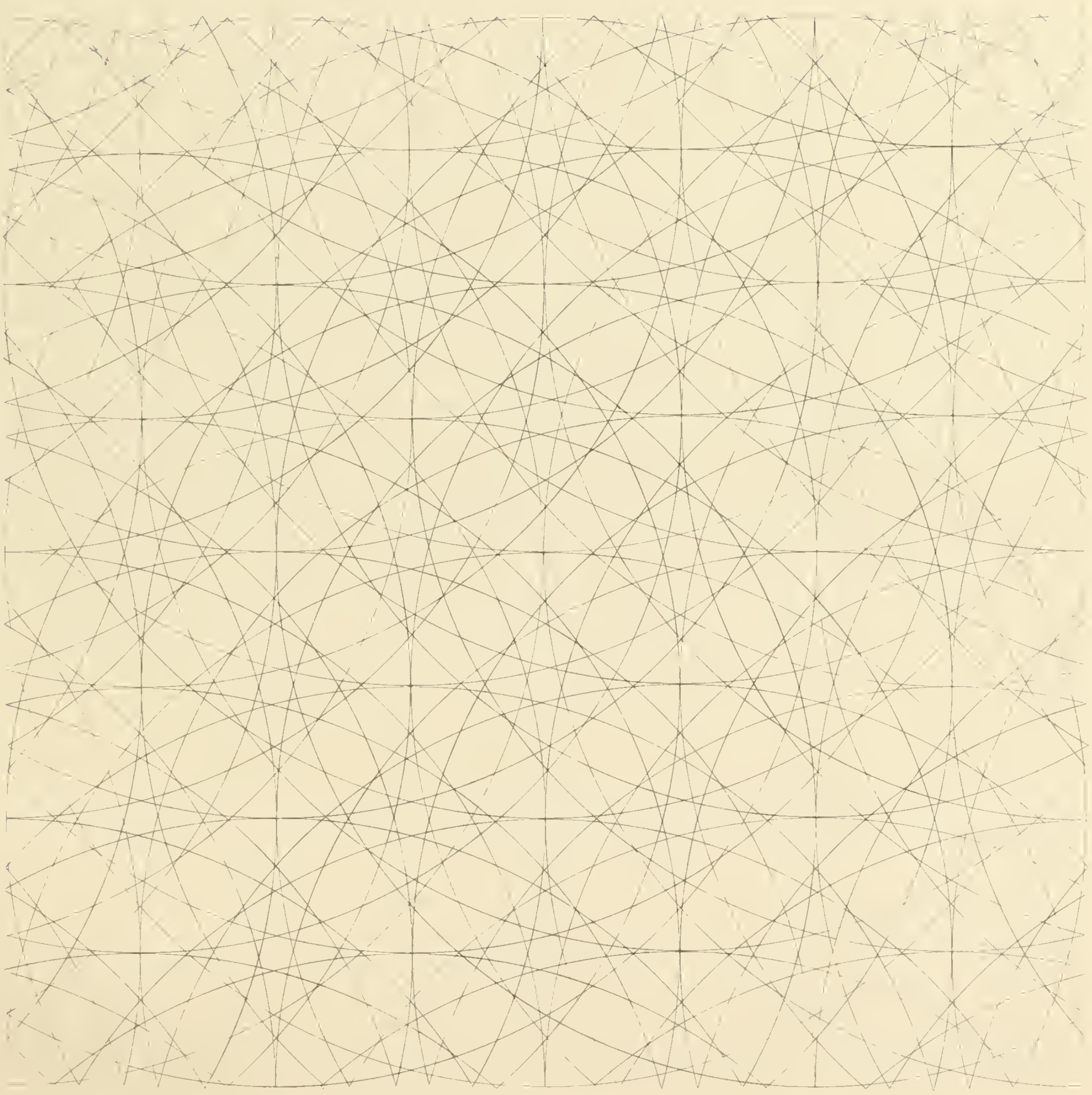
OF THE HARMONIC RATIOS OF NUMBERS.

The unit has no power of multiplication or division, but the second numeral 2 has both those powers, and they are productive in both

* The Author has in progress, and shortly intends to publish, a series of Diagrams upon those principles, with the diaper designs resulting from them.

Plate IX.

A Diagram composed of the Homogeneous Curve.



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Fr. Schenck Lith. Edin.r

PLATE X.

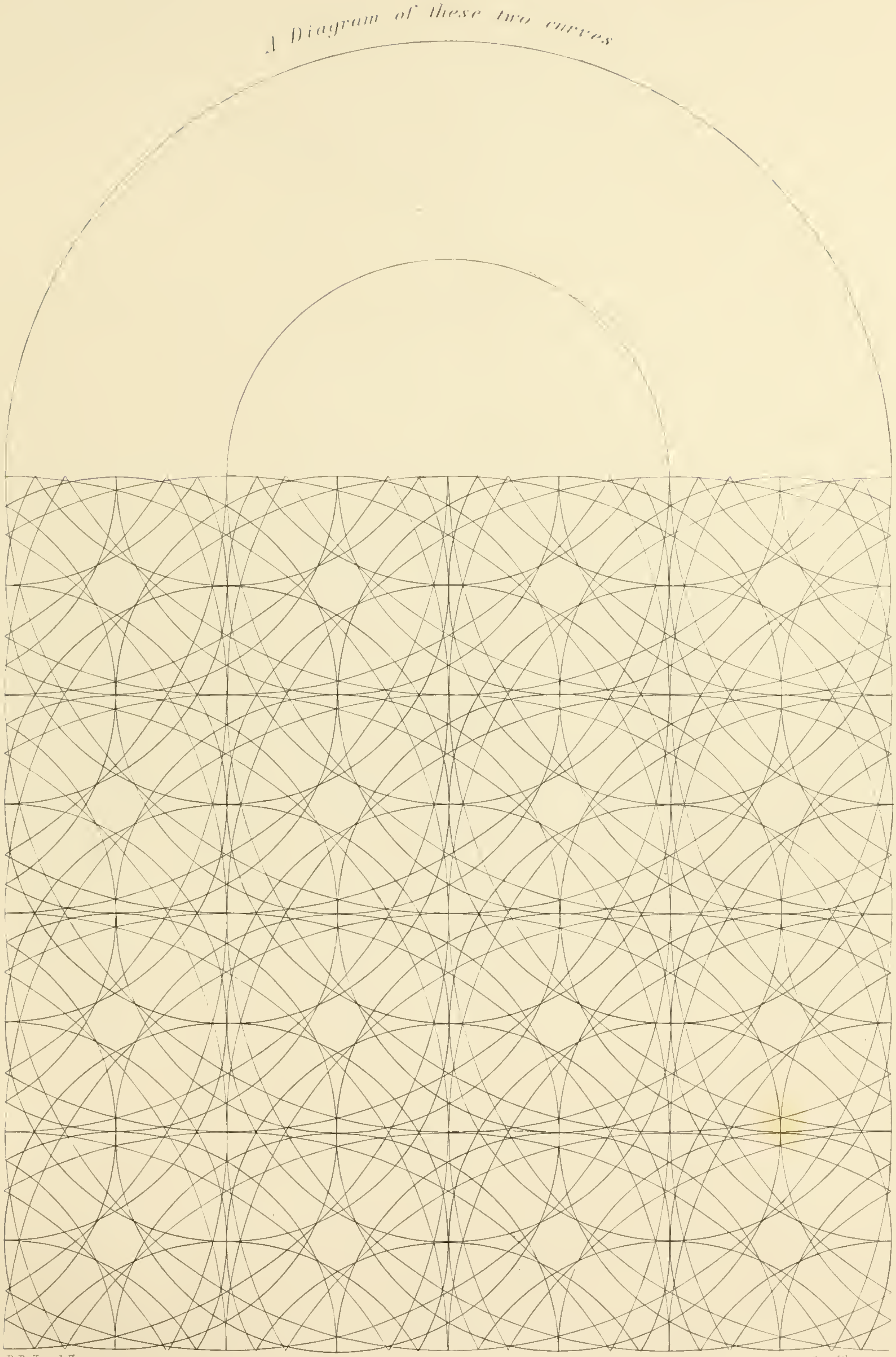

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PLATE XI.

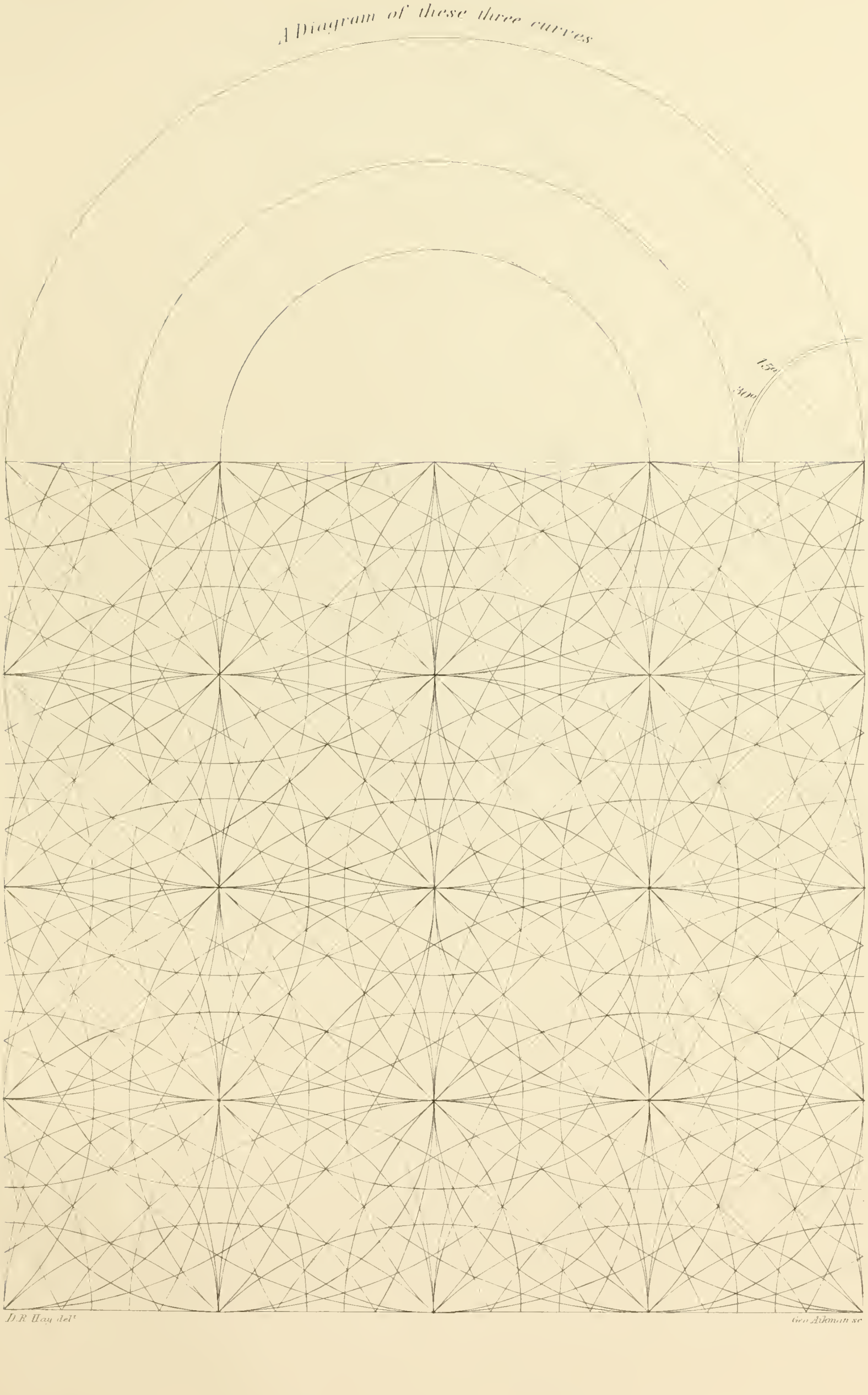

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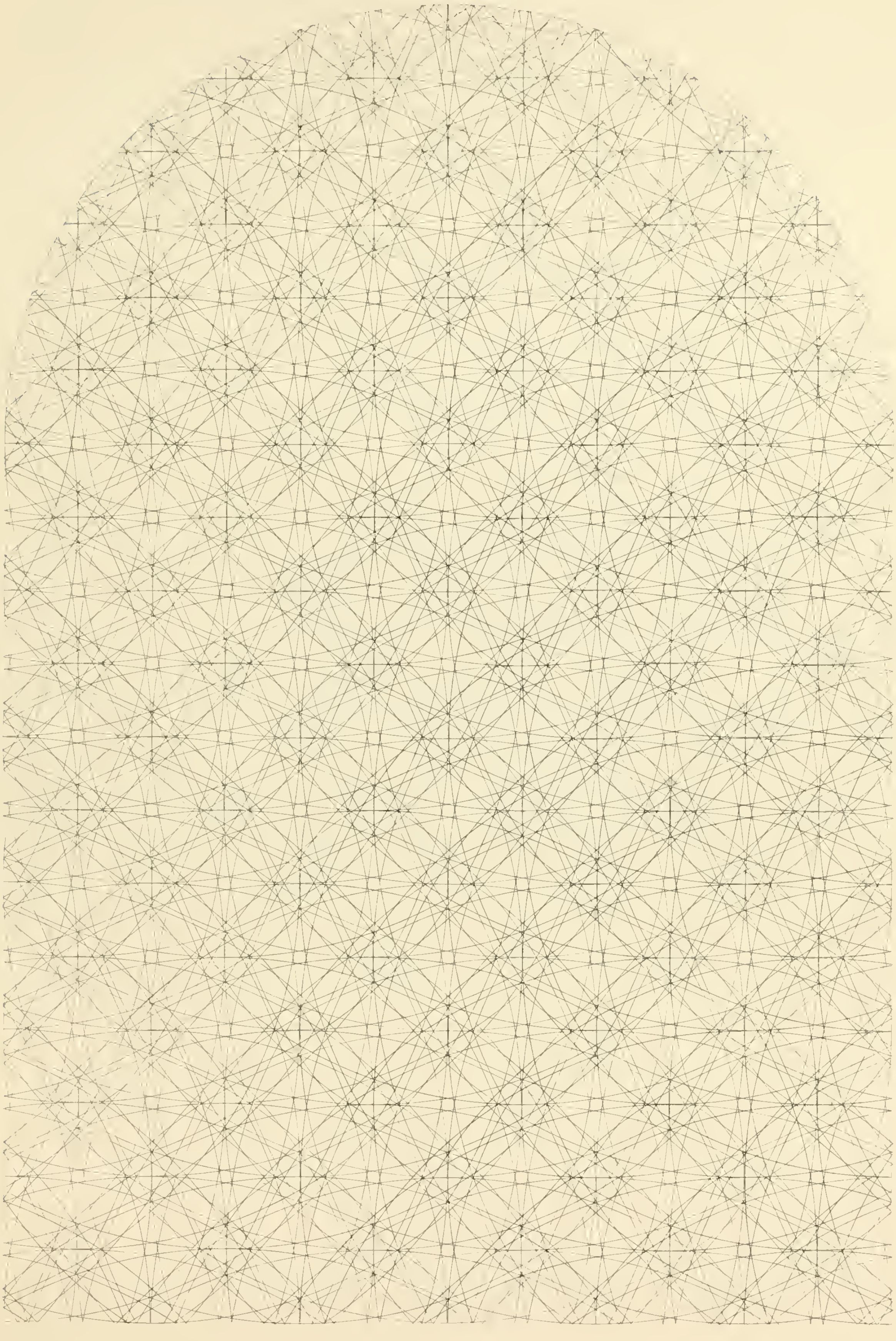
PLATE XII.

A Diagram composed of the Homogeneous Curve.



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Engrᵈ on stone by Fr. Schenck, Lith. Edinʳ

cases of uniformity. It is the first multiple of 1. It is a submultiple of the numbers 4, 6, 8, &c., progressively, as 2, 3, 4; and it is the first even number. The second numeral 3 is also simply a multiple of 1, and is the first odd number; it is a submultiple of 6, 9, 12, &c., progressively, as 2, 3, 4, and is a compound of 1 and 2 added together.

The next multiple of 1, having no other aliquot parts, is 5—a similar compound of the first even and first odd numbers, 2 and 3. It is a submultiple of 10, &c. These three numerals 2, 3, and 5, are, therefore, the first three multiples of 1 that are multiples of no other number—consequently they are adapted to divide the elements of proportion into the primary harmonic ratios, as already shown; and in this capacity equally regulate the effects produced by external nature upon the senses of hearing and seeing. The number 5, although by it the third mode of division is performed, is of an intermediate character, and the rectangle produced by its division of the quadrant, is the mediant in the series, as already shown: the middle may thus be said to be produced by a number which combines the first and last.

The primary harmonic ratios produced by those modes of division, are, as already shown, 1 to 2, 2 to 3, 4 to 5, and the secondary ones being 8 to 9, 3 to 4, 3 to 5, and 8 to 15, no new mode of division is required, 4, 8, 9, and 15, being multiples of 2, 3, and 5.* To arrange and proportion the combination of various forms by those, in such a manner as to produce one or unity, ought on all occasions to be the aim of the artist; for every composition should manifest in all its parts a definite relation to a whole. This is the first condition of order, and consequently the primary cause of geometric beauty.

When the parts of a figure are perfectly homogeneous or identical, there can exist in it no principle of proportion productive of this effect, because its parts are throughout as 1 to 1, producing sameness. The circle, the square, and the equilateral triangle, are figures of

* The number 7 is not employed to produce the harmonic ratios; but if the quadrant be divided by 18, and the product multiplied by 7, it will give 35°; and a rectangle formed upon this diagonal will have the peculiar property of being divided into two of its own proportions by a line bisecting it through its shortest diameter.

this description, and consequently want within themselves individually the first element of proportion—variety. It would appear that those figures can only be proportioned individually in octaves in regard to superficies, and in octaves and fifths in regard to periphery. Therefore, two squares are in proportion, if one be either in area or perimeter to the other, as 1 to 2, as shown on Plate XIII., Figure 1, where A to B, B to C, C to D, and D to E, will be found to be respectively in the ratio of 1 to 2 in quantity of area or superficies, while in perimeter they are as 2 to 3. But A to C, B to D, and C to E, will be found in the lineal quantity of their perimeter in the ratio of 1 to 2.

The circles inscribing and inscribed by those squares will necessarily have the same binary and ternary ratios, Plate XIII., Figure 1; but the simple mode of division shown by the strong lines in Figure 3, seems the most harmonious.

The equilateral triangle cannot harmonically inscribe a larger figure of its own kind than that which bears to it the superficial ratio of 1 to 4, a double octave. A is to B, and B to C, in that ratio, and are in perimeter in that of 1 to 2, as shown on Plate XIII., Figure 2. This seems to be the only kind of proportion that can be applied to those figures while they retain their homogeneous quality.

The secondary figures—the oblong, the lozenge, and the ellipse—may also be divided in this manner, and the ratios in the quantity of their superficies, perimeter, or circumference, under every degree of modification accurately ascertained, as exemplified on Plate XIII., Figures 4, 5, 6. Figure 7 is the hexagon, with its peripheral octave, which latter figure will be found in Figure 3 (D) harmoniously inscribed in the equilateral triangle, and bearing to it in quantity of area the relative proportion of 1 to 2.

But those secondary figures have also, in the parts of which they are composed, that first requisite in proportion—variety. The harmonic ratios are therefore applicable to those figures individually, and their beauty will depend upon the proportioning of their length to their breadth.

The oblong I have adopted as the leading figure of the secondary

PLATE XIII.

The seven elements of Geometric Beauty.

Fig. 1.

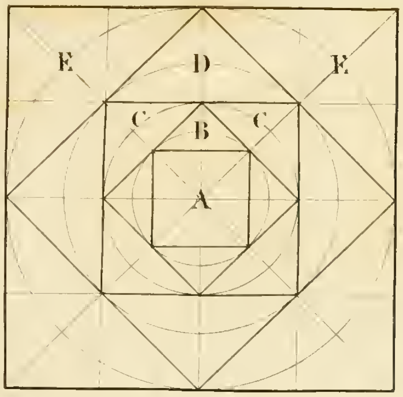


Fig. 2.

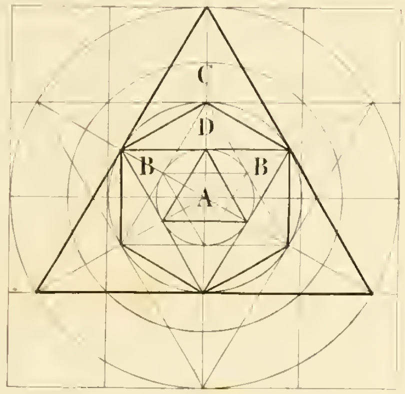


Fig. 3.

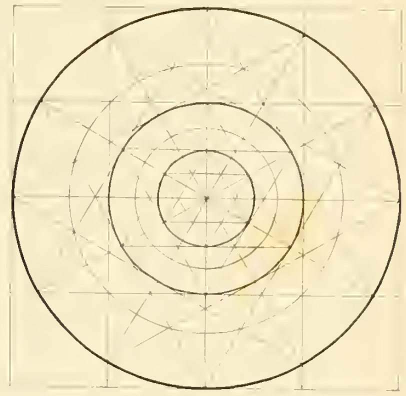


Fig. 4.

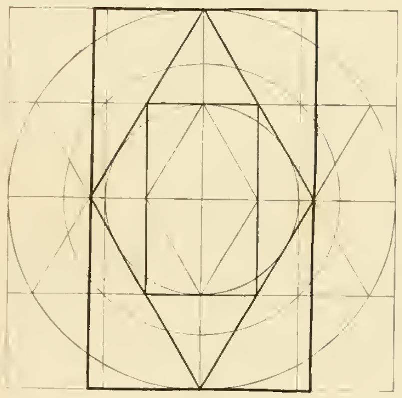


Fig. 5.

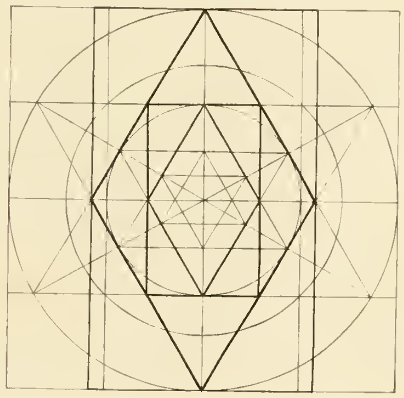


Fig. 6.

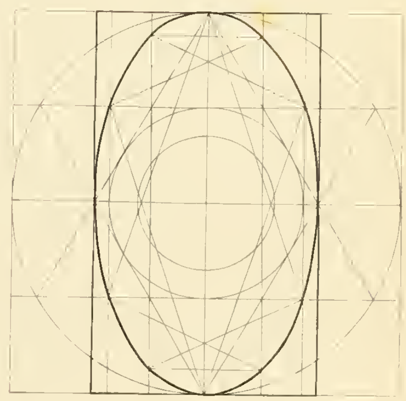
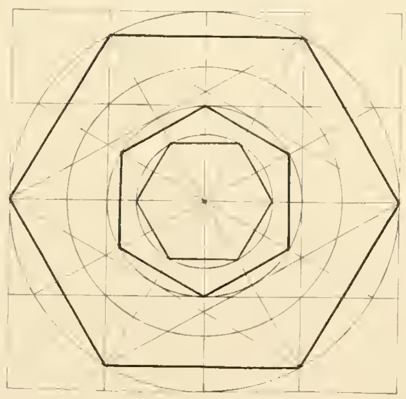


Fig. 7.



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class, has peculiarities in its proportions which are worthy of remark. They are as follow:—Its vertical diagonal is 60°, and its horizontal diagonal 30°, consequently its proportions are, in this respect, in the most consonant harmonic ratio of 1 to 2. It may, in common with all other figures of this class, be divided, by two lines across its area, into four of its own proportions. But it may also, by two lines drawn across its shortest diameter, be divided into three figures of its own proportions. If divided into two diagonally, it produces two scalene triangles, which, on being joined by the longest side of the right angle, form an equilateral triangle. Its vertical diagonal is to the right angle in the harmonic ratio of 2 to 3, and its horizontal diagonal bears the same harmonic ratio to the diagonal of the primary square. Those proportions render this figure more intrinsically symmetrical, and consequently more pleasing to the eye, than any other of its own class.

The ellipse and rhombus, adopted as the leading figures of the classes to which they respectively belong, are proportioned to this figure, and consequently partake of its beauty.

It has already been stated, that, by a reciprocity existing between a rectangle and the curvilinear figure which it inscribes, the lines by which the area of the former is divided into sixteen equal homogeneous parts, will, at the points of intersection, harmoniously divide the circumference of the latter into twelve, and that, by uniting those points by diagonal lines drawn across the area of the inscribed circle or ellipse, the angles in harmony with the inscribing rectangle will be produced. The three leading rectangles, with their curvilinear and angular figures, are given in Plate XIV., on which the coincidences of the intersections are very remarkable.

In Plate XV., I give the various triangles belonging to each of the three leading rectangles, where it will be seen that the circle has on its lowest chord the equilateral, [figure 1;] on its centre chord the right-angled isosceles, [figure 2;] and on the upper the obtuse-angled isosceles, [figure 3.] The harmonic ratios they bear to the right angle are those of a fifth, an octave, and a twelfth, their degrees of obliquity being 60°, 45°, 30°. The first ellipse has, upon its lowest chord, the right-angled isosceles, [figure 4]—upon its centre chord,

PLATE XIV.

A Diagram of the Three series of figures produced by the division of the quadrant by 2, by 3, and by 5, showing their vertical and horizontal relations.

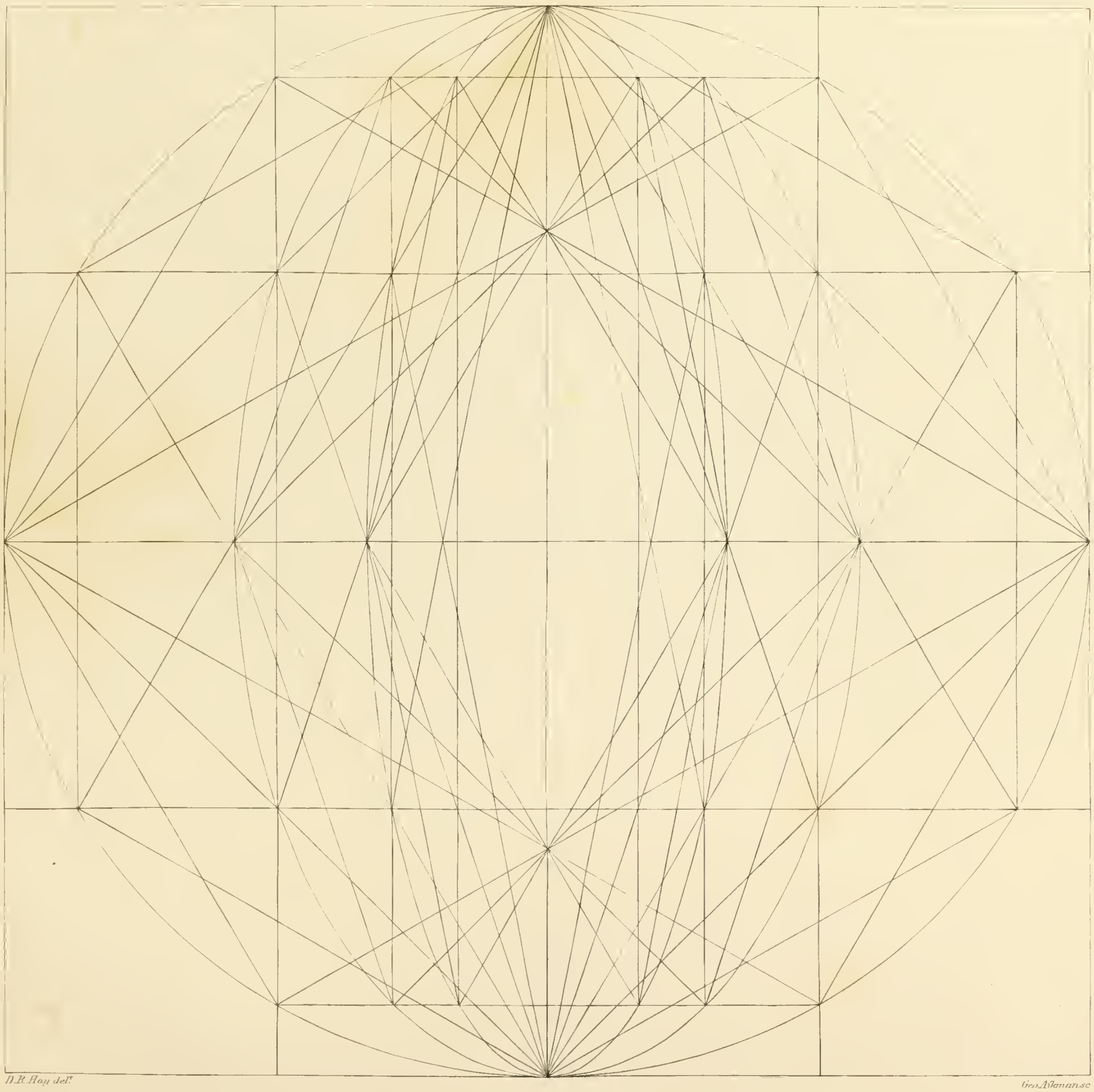

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PLATE XV.

Fig. 7.

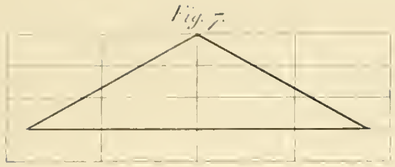


Fig. 8.

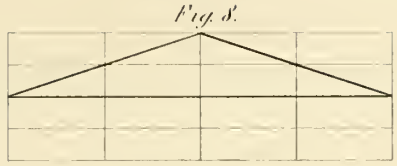


Fig. 9.

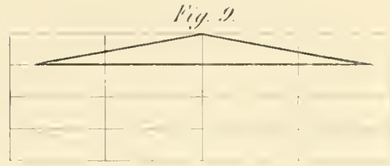


Fig. 4.

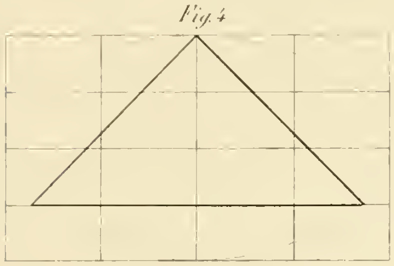


Fig. 5.

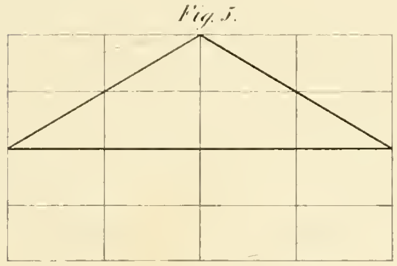


Fig. 6.

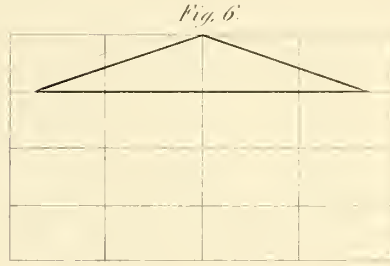


Fig. 1.

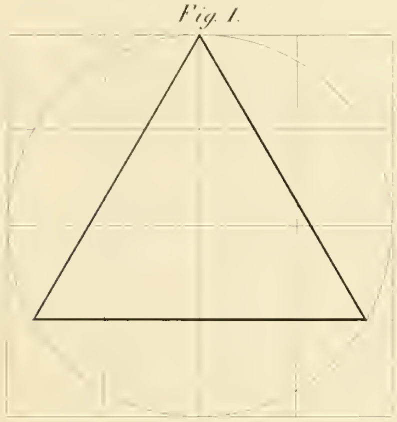


Fig. 2.

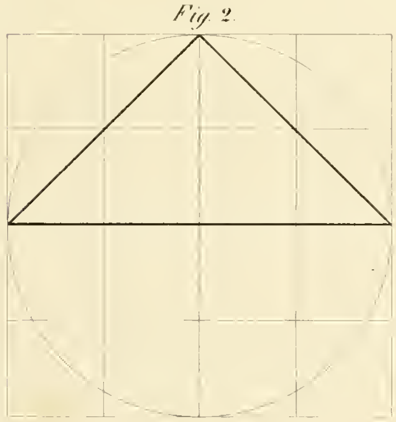


Fig. 3.

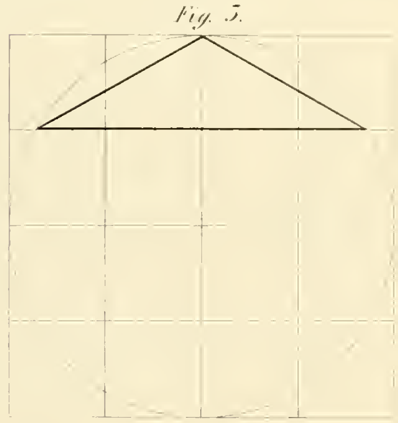


Fig. 10.

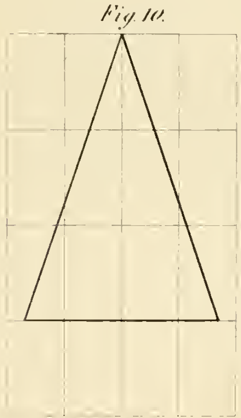


Fig. 11.

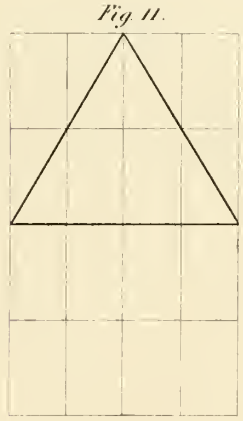


Fig. 12.

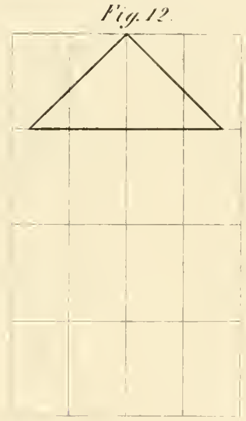


Fig. 13.

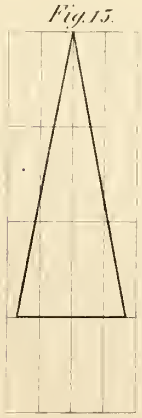


Fig. 14.

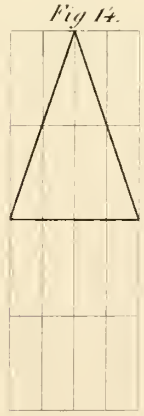


Fig. 15.

D. R. Hay del^t. Fr. Schenck Lith. 9 Greenside Pl.

the obtuse-angled isosceles, [figure 5]—and, upon its upper chord, the second obtuse-angled isosceles, [figure 6.] The harmonic ratios of those angles are to the right angle those of an octave, a twelfth, and a seventeenth, being 45°, 30°, 18°. The second ellipse has, upon its lowest chord, the obtuse-angled isosceles, [figure 7]—upon its centre chord the same figure of the second class, [figure 8]—and, upon its upper chord, the same figure of the third class, [figure 9.] Their harmonic ratios are to the right angle those of a twelfth, a seventeenth, and a twenty-second, or treble octave, being 30°, 18°, 11° 15'. The vertical triangles of the two ellipses are given in Figures 10, 11, 12, 13, 14, and 15.

Thus have I endeavoured to analyse the Geometric principle of Beauty—proportion—by showing that it is regulated by the harmonic ratios of numbers. And by the application of those ratios to a quadrant of the circle, I have shown that an almost infinite series of rectangles may be produced, bearing to one another certain harmonious relations; and that within each of those a series of six other distinctive characters of figures may be systematically and harmoniously generated. In short, that the beauty arising from the harmony of form may be, on all occasions, with certainty produced.

But the application of this system to the various arts in which it will be useful, must form the subject of another treatise, as it would be premature to apply rules until their accuracy were acknowledged. I shall, however, in the mean time, add a few general rules which obviously arise out of this theory :—

1st, Rectangles, when arranged in succession, either horizontally or vertically, should only differ from one another in one of their dimensions ; so that, when vertically arranged either as solids or vacuities,* their vertical sides must be in the same line ; and when horizontally arranged, their horizontal lines must also be in the same line, their

* Professor Hosking, in his excellent Treatise on Architecture, amongst other valuable observations upon the harmony of form as applicable to that art, says, that " a degree of harmony must exist between the solids and vacuities of an edifice." This I consider to form, along with the outline, the melody or leading feature in all such compositions ; for if it be inharmonious, no profusion of ornament can conceal the deformity.

harmony being regulated by their diagonals alone. See Plates II. and III.; as also Plates XVI. and XVII.

2d, Triangles must on all occasions correspond to the rectangles with which they are associated—acutely when the rectangle is vertical, and obtusely when it is horizontal. As the harmonious proportion of every rectangle, when in a vertical or horizontal position, is determined by an oblique line called its diagonal, so is the proportion of every regular isosceles triangle determined by a vertical line. This being in a positive position, it can have no change but in its lineal dimensions, which will be seen in Plate XV. to be always as 1 to 2, or 2 to 3. When, therefore, triangles are employed in succession, their proportions must be in other respects the same. If they be not, they generate between them a discordant figure.

Curvilinear *figures*, in like manner, must always correspond to the rectangles with which they are associated, and in succession their harmony will depend upon the ratio of their radii. Therefore, they can only differ in size, and not in degree of curvature. This difference can only be in the ratio of 1 to 2, or 2 to 3. The curve can never be greater than what may be inscribed by the rectangle with which it is associated, and can never harmoniously leave the rectangle unless at the tangential point, or at right angles with it. Plate XV.

As a circle may be described within any rectangle tangential to its longest sides, this peculiar curve may terminate any vertical rectangle.

Plates XVI. and XVII. are examples of harmonic combinations of rectangles, divided into triangles agreeably to the Platonic system.* The strong lines show how they may be formed into solids and vacuities in architectural composition. These very simple examples are added, merely to show the mode in which the harmonic angles may be applied.

* See Appendix.

APPENDIX.

The careful and learned notice of my former treatise on this subject, to which I have already alluded, and which appeared when the greater portion of the present Essay was printed, has not only convinced me of the accuracy of what I have stated as likely to have conduced to the perfection of the arts in Greece, but has given me a key to the elucidation of my subject. The author of the notice alluded to, after having pointed out with critical distinctness the errors I had committed in that treatise in regard to analogy, and to point out the true cause of harmony of form, states—

"We now proceed to show that the principles we have previously developed as the Greek doctrines of beauty and proportion, give us precisely Mr Hay's elements, along with many others which his system excludes. They are given in the Timæus of Plato."*—*Athenæum*, No. 817, p. 586.

From this I was induced to examine the Platonic Theory, and shall here quote the whole of what is in any way applicable either to my last treatise or to the present attempt. This, I trust, will enable those who, like myself, are not conversant with the works of the ancient philosophers, to determine in how far I have now succeeded in reducing to system the elements alluded to.

"But when the Artificer began to adorn the universe, he first of all figured with forms and num-

* In another learned review of my former treatise, the writer says :—"It is, after all, the developed theory of Plato, who eloquently commented on the music of beautiful forms. * * It is a startling proof of the depth of the Egypto-Platonic, or geometrical philosophy, transmitted to the Templar Freemasons."—*Court Gazette*, No. 268, p. 195.

bers fire and earth, water and air, which possessed, indeed, certain traces of the true elements, but were in every respect so constituted, as it becomes any thing to be from which Deity is absent. But we should always persevere in asserting that Divinity rendered them as much as possible the most beautiful and the best, when they were in a state of existence opposite to such a condition. I shall now, therefore, endeavour to unfold to you the distribution and generation of these by a discourse, unusual indeed, but to you who have trod in all the paths of erudition through which demonstration is necessarily obtained, perspicuous and plain. In the first place, then, that fire and earth, water and air, are bodies, is perspicuous to every one. But every species of body possesses profundity; and it is necessary that every depth should comprehend the nature of a plane. Again, the rectitude of the base of a plane is composed from triangles. But all triangles originate from two species; one of which possesses one right angle, and the other two acute angles: and one of these contains one right angle distributed with equal sides; but, in the other, unequal angles are distributed with unequal sides. Hence, proceeding according to assimilative reasons, conjoined with necessity, we shall establish a principle of this kind as the origin of fire and all other bodies. The supernal principles of these, indeed, are known to Divinity, and to the man who is in friendship with Divinity.

"But it is necessary to relate by what means four most beautiful bodies were produced; dissimilar indeed to each other, but which are able, from certain dissolutions into each other, to become the sources of each other's generation. For if we are able to accomplish this, we shall obtain the truth concerning the generation of earth and fire, and of those elements which are situated according to analogy between these. And then we shall not assent to any one who should assert that there are visible bodies more beautiful than these, each of which subsists according to one kind. We must endeavour, therefore, to harmonize the four sorts of bodies excelling in beauty, and to evince by this means that we sufficiently comprehend the nature of these. Of the two triangles, indeed, the isosceles is allotted one nature, but the oblong or scalene is characterized by infinity. We ought, therefore, to choose the most beautiful among infinities, if we wish to commence our investigation in a becoming manner. And if any one shall assert that he has chosen something more beautiful for the composition of these, we shall suffer his opinion to prevail; considering him not as an enemy, but as a friend. Of many triangles, therefore, we shall establish one as most beautiful, (neglecting the rest;) I mean the equilateral, which is composed from three parts of a scalene triangle. To assign the reason of this would, indeed, require a prolix dissertation; but a pleasant reward will remain for him who, by a diligent investigation, finds this to be the case. We have, therefore, selected two triangles out of many, from which the body of fire and of the other elements is fabricated; one of which is isosceles, but the other is that which always has its longer side triply greater in power than the shorter.

"But that which we formerly asserted without sufficient security, it is now necessary more accurately to define. For it appeared to us, though improperly, that all these four natures were mutually generated from each other; but they are in reality generated from the triangles which we have just described—three of them, indeed, from one triangle containing unequal sides; but the fourth alone is aptly composed from the isosceles triangle. All of them, therefore, are not able, by a dissolution into each other, to produce from many small things a mighty few, or the contrary. This, indeed, can be effected by three of them; for, as all the three are naturally generated from one triangle, when the greater parts are dissolved, many small parts are composed from them, receiving figures accommodated to their natures. And, again, when the many small parts, being scattered according to triangles, produce a number of one bulk, they complete one mighty species of a different kind. And thus much may suffice concerning their mutual generation."

"It now remains that we should speak concerning the quality of each of their kinds, and relate from what concurring numbers they were collected together. The first species, indeed, is that which was composed from the fewest triangles, and is the element of that which has its longer side twice the length of the shorter side, which it subtends. But two of these being mutually placed according to the diameter, and this happening thrice, the diameters and the shorter sides passing into the same as into a centre, hence one equilateral triangle is produced from six triangles. But four equilateral triangles being composed according to three plane angles, form one solid angle; and this the most obtuse of all the plane angles from which it is composed. Hence, from four triangles of this kind receiving their completion, the first solid species was constituted, distributive of the whole circumference into equal and similar parts. But the second was formed from the same triangles, but at the same time constituted according to eight equilateral triangles, which produced one solid angle from four planes ; so that the second body received its completion from the composition of six triangles of this kind. And the third arose from the conjunction of twice sixty elements, and twelve solid angles, each of which having twenty equilateral bases, is contained by five plane equilateral triangles. In this manner, then, the other elements generated these. But the isosceles triangle, being constituted according to four triangles, and collecting the right angles at the centre, and forming one equilateral quadrangle, generated the nature of the fourth element. But six such as these being conjoined, produced eight solid angles, each of which is harmonized together, according to three plane right angles. Hence the figure of the body thus composed is cubical, obtaining six plane quadrangular equilateral bases. There is also a certain fifth composition, which Divinity employed in the fabrication of the universe, and when he delineated those forms, the contemplation of which may justly lead some one to doubt whether it is proper to assert that the number of worlds is infinite or finite ;—though, indeed, to affirm that there are infinite worlds, can only be the dogma of one who is ignorant about things in which it is highly proper to be skilful. But it may with much less absurdity be doubted whether there is in reality but one world, or whether there are five. According to our opinion, indeed, which is founded on assimilative reasons, there is but one world ; though some one, regarding in a certain respect other particulars, may be of a different opinion. But it is proper to dismiss any further speculations of this kind."—*The Works of Plato, translated by Taylor,* Vol. II. p. 526.

The writer in the *Athenæum,* already quoted,* thus explains and simplifies what is said in the above quotation regarding triangles :—

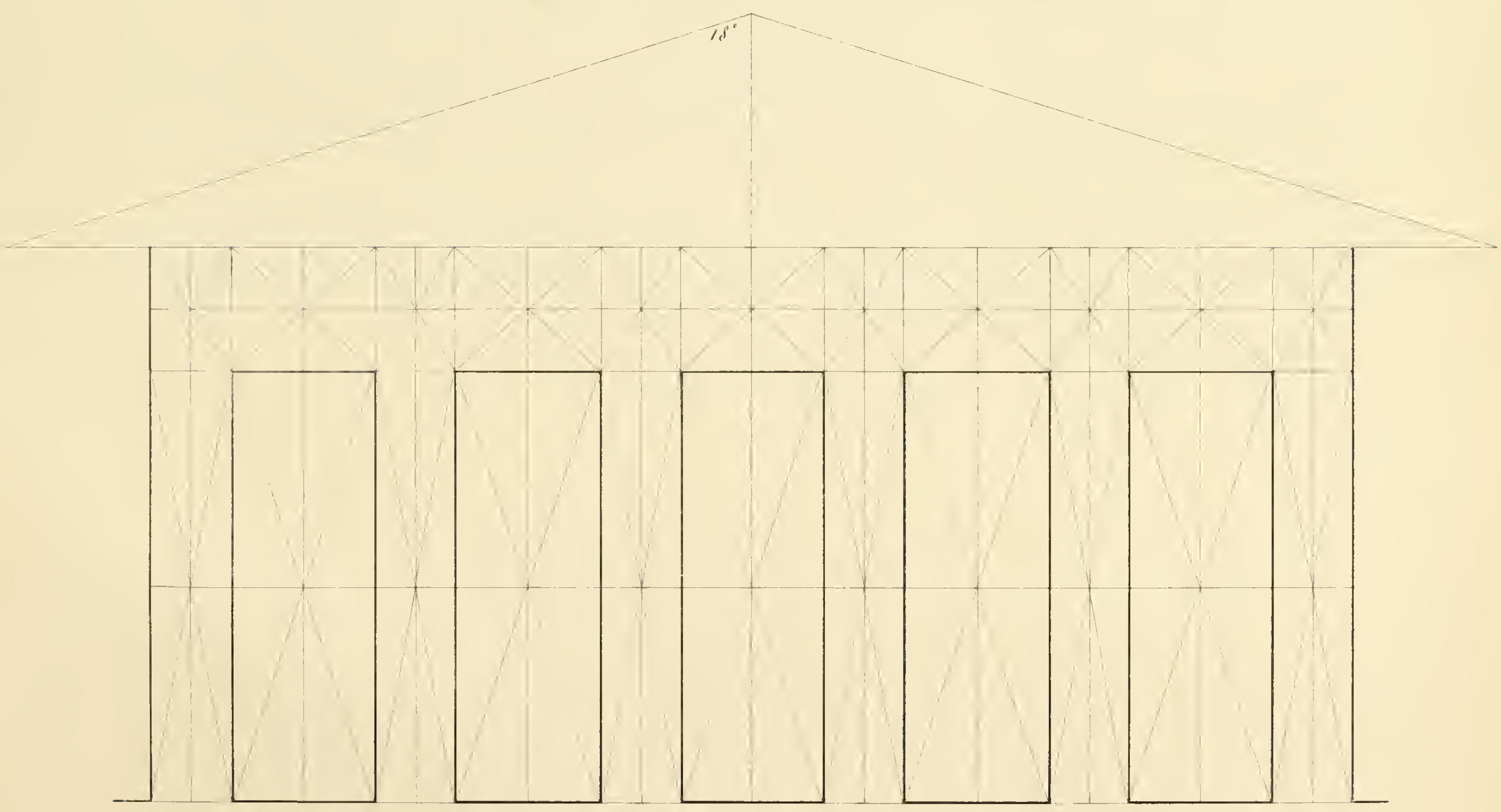
" The two symmetrical triangles of Plato are—1, that triangle which is half of the square, and, 2, that which is half of the equilateral triangle." * * * " The first exemplifies the relations of the number 2, and the second Platonic triangle adds to these the properties of the number 3."

* No. 817, p. 586.

Plate XVI.

A composition of Rectangles formed upon the diagonals of 80° 72° 60° and 42°. The three first are in the harmonic ratios to the right angle of 8 to 9, 4 to 5 and 2 to 3 and 42° being the horizontal of 48° is in the ratio of 7 to 15.

The above Rectangles divided into scalene triangles agreeably to the Platonic Theory of the elements, and surmounted by the horizontal of 72° which has an angle of 18° and is consequently in the harmonic ratio to the right angle of 1 to 5.

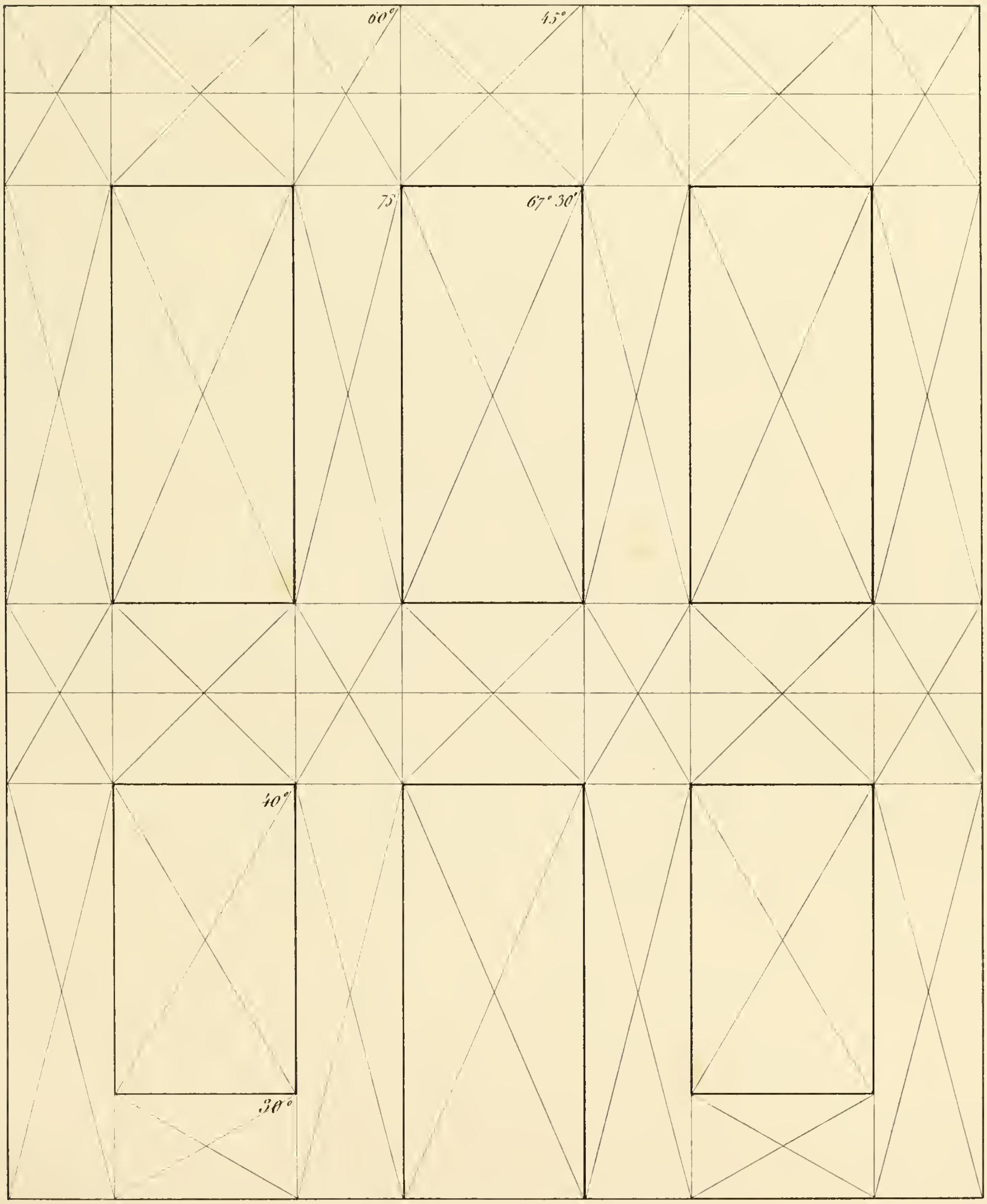


D. R. Hay del.t

Schenck, lith. 8 Greens Pl.

Plate XVII.

A composition of Rectangles formed upon the diagonals of 75° 67° 30', 60° 45° and 30°, having the harmonic ratio to the right angle of 5 to 6, 3 to 4, 2 to 3, 1 to 2 and 1 to 3.



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This is precisely the situation in which these two angles occur upon the division of the quadrant by the numbers 2 and 3, namely, 45° and 60°, which have been shown to be in the harmonic ratio to the right angle of 1 to 2 in the first instance, and of 2 to 3 in the second. Those angles were, therefore, adopted by me as the tonic and dominant of my present system, long before I was aware of the high authority by which this fundamental part of my theory is supported.

It seems clear, however, that the mediant angle, as well as the tonic and dominant, must have been known to Plato; for he mentions a triangle "*which always has its longest side triply greater in power than the shorter.*" Now, this triangle is neither the half of the square nor of the equilateral triangle; but if the quadrant be divided by five, a triangle of this description is the result, and this completes the primaries.

Fig. 1.

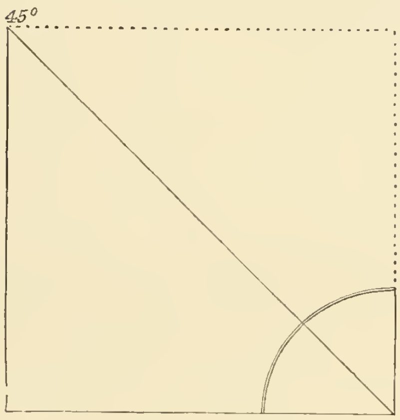


Fig. 2.

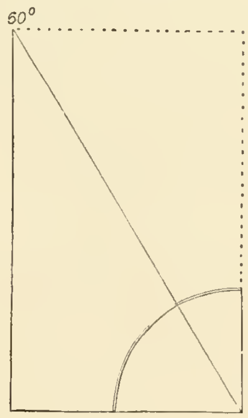
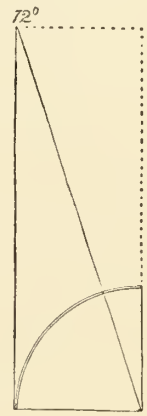


Fig. 3.



I therefore consider this authority quite conclusive of the fact, that the perfection of the arts in Greece was the result of a system of proportion similar to what I have endeavoured to develop in the foregoing pages.

All the combinations of the harmonic angles that can be treated of in an essay of this kind, are only such as may be depicted upon the retina—namely, plane polygons. Therefore the combination of the solid triangles, producing the polyhedrons which Plato conceived to form the atomic particles of the four elements, can in no way apply to the object of this treatise; while, on the other hand, it includes numerous plane figures unnoticed in the Platonic Theory of triangles.

THE END.

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ERRATA.

Page 29, last line, *for* second, *read* secondary.

Pages 38 and 48, *for* harmonies, *read* harmonics.

Page 66, 9th line from bottom, *for* figure 3, *read* figure 2.

HAY

ON

PROPORTION.