THE

SCIENCE OF BEAUTY,

AS DEVELOPED IN NATURE AND APPLIED IN ART.

BY

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"The irregular combinations of fanciful invention may delight awhile, by that novelty of which the common satiety of life sends us all in quest; the pleasures of sudden wonder are soon exhausted, and the mind can only repose on the stability of truth." — DR JOHNSON.

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TO

JOHN GOODSIR, ESQ., F.R.SS.L.&E.,

PROFESSOR OF ANATOMY IN THE UNIVERSITY OF EDINBURGH,

AS AN EXPRESSION OF GRATITUDE FOR VALUABLE ASSISTANCE,

AS ALSO OF HIGH ESTEEM AND SINCERE REGARD,

THIS VOLUME IS DEDICATED,

BY

THE AUTHOR.

PREFACE.

My theory of beauty in form and colour being now admitted by the best authorities to be based on truth, I have of late been often asked, by those who wished to become acquainted with its nature, and the manner of its being applied in art, which of my publications I would recommend for their perusal. This question I have always found difficulty in answering; for although the law upon which my theory is based is characterised by unity, yet the subjects in which it is applied, and the modes of its application, are equally characterised by variety, and consequently occupy several volumes.

Under these circumstances, I consulted a highly respected friend, whose mathematical talents and good taste are well known, and to whom I have been greatly indebted for much valuable assistance during the course of my investigations. The advice I received on this occasion, was to publish a *résumé* of my former works, of such a character as not only to explain the nature of my theory, but to exhibit to the general reader, by the most simple modes of illustration and description, how it is developed in nature, and how it may be extensively and with ease applied in those arts in which beauty forms an essential element.

The following pages, with their illustrations, are the results of an attempt to accomplish this object.

To those who are already acquainted, through my former works, with the nature, scope, and tendency of my theory, I have the satisfaction to intimate that I have been enabled to include in this *résumé* much original matter, with reference both to form and colour.

D. R. HAY.

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INTRODUCTION.

Twelve years ago, one of our most eminent philosophers,* through the medium of the *Edinburgh Review*,† gave the following account of what was then the state of the fine arts as connected with science:—"The disposition to introduce into the intellectual community the principles of free intercourse, is by no means general; but we are confident that Art will not sufficiently develop her powers, nor Science attain her most commanding position, till the practical knowledge of the one is taken in return for the sound deductions of the other. . . . . . It is in the fine arts, principally, and in the speculations with which they are associated, that the controlling power of scientific truth has not exercised its legitimate influence. In discussing the principles of painting, sculpture, architecture, and landscape gardening, philosophers have renounced science as a guide, and even as an auxiliary; and a school has arisen whose speculations will brook no restraint, and whose decisions stand in opposition to the strongest convictions of our senses. That the external world, in its gay colours and lovely forms, is exhibited to the mind only as a tinted mass, neither within nor without the eye, neither touching it nor distant from it—an ubiquitous chaos, which experi-

* Sir David Brewster. † No. CLVIII., October 1843.

ence only can analyse and transform into the realities which compose it; that the beautiful and sublime in nature and in art derive their power over the mind from association alone, are among the philosophical doctrines of the present day, which, if it be safe, it is scarcely prudent to question. Nor are these opinions the emanations of poetical or ill-trained minds, which ingenuity has elaborated, and which fashion sustains. They are conclusions at which most of our distinguished philosophers have arrived. They have been given to the world with all the authority of demonstrated truth; and in proportion to the hold which they have taken of the public mind, have they operated as a check upon the progress of knowledge."

Such, then, was the state of art as connected with science twelve years ago. But although the causes which then placed science and the fine arts at variance have since been gradually diminishing, yet they are still far from being removed. In proof of this I may refer to what took place at the annual distribution of the prizes to the students attending our Scottish Metropolitan School of Design, in 1854, the pupils in which amount to upwards of two hundred. The meeting on that occasion included, besides the pupils, a numerous and highly respectable assemblage of artists and men of science. The chairman, a Professor in our University, and editor of one of the most voluminous works on art, science, and literature ever produced in this country, after extolling the general progress of the pupils, so far as evinced by the drawings exhibited on the occasion, drew the attention of the meeting to a discovery made by the head master of the architectural and ornamental department of the school, viz.—That the ground-plan of the Parthenon at Athens had been constructed by the application of the *mysterious* ovoid or *Vesica*

Piscis of the middle ages, subdivided by the *mythic* numbers 3 and 7, and their intermediate odd number 5. Now, it may be remarked, that the figure thus referred to is not an ovoid, neither is it in any way of a mysterious nature, being produced simply by two equal circles cutting each other in their centres. Neither can it be shewn that the numbers 3 and 7 are in any way more mythic than other numbers. In fact, the terms *mysterious* and *mythic* so applied, can only be regarded as a remnant of an ancient terminology, calculated to obscure the simplicity of scientific truth, and when used by those employed to teach—for doubtless the chairman only gave the description he received—must tend to retard the connexion of that truth with the arts of design. I shall now give a specimen of the manner in which a knowledge of the philosophy of the fine arts is at present inculcated upon the public mind generally. In the same metropolis there has likewise existed for upwards of ten years a Philosophical Institution of great importance and utility, whose members amount to nearly three thousand, embracing a large proportion of the higher classes of society, both in respect to talent and wealth. At the close of the session of this Institution, in 1854, a learned and eloquent philologus, who occasionally lectures upon beauty, was appointed to deliver the closing address, and touching upon the subject of the beautiful, he thus concluded—

"In the worship of the beautiful, and in that alone, we are inferior to the Greeks. Let us therefore be glad to borrow from them; not slavishly, but with a wise adaptation—not exclusively, but with a cunning selection; in art, as in religion, let us learn to prove all things, and hold fast that which is good—not merely one thing which is good, but all good things—Classicalism, Mediævalism, Modernism—let us have and hold them all in one wide and lusty embrace. Why should the world of art be more narrow, more monotonous, than the world of nature? Did God make all the flowers of one pattern, to please the devotees of the rose or the lily; and did He make all the hills, with the green folds of their queenly mantles, all at one slope, to suit the angleometer of the most mathematical of decorators? I trow not. Let us go and do likewise."

I here take for granted, that what the lecturer meant by "the worship of the beautiful," is the production and appreciation of works of art in which beauty should be a primary element; and judging from the remains which we possess of such works as were produced by the ancient Grecians, our inferiority to them in these respects cannot certainly be denied. But I must reiterate what I have often before asserted, that it is not by borrowing from them, however cunning our selection, or however wise our adaptations, that this inferiority is to be removed, but by a re-discovery of the science which these ancient artists must have employed in the production of that symmetrical beauty and chaste elegance which pervaded all their works for a period of nearly three hundred years. And I hold, that as in religion, so in art, there is only one truth, a grain of which is worth any amount of philological eloquence.

I also take for granted, that what is meant by Classicalism in the above quotation, is the ancient Grecian style of art; by Mediævalism, the semi-barbaric style of the middle ages; and by Modernism, that chaotic jumble of all previous styles and fashions of art, which is the peculiar characteristic of our present school, and which is, doubtless, the result of a system of education based upon plagiarism and mere imitation. Therefore a recommendation to embrace with equal fervour "as good things," these very opposite arti*cisms* must be a doctrine as mischievous in art as it would be in religion to recommend as equally good things the various *isms* into which it has also been split in modern times.

Now, "the world of nature" and "the world of art" have not that equality of scope which this lecturer on beauty ascribes to them, but differ very decidedly in that particular. Neither will it be difficult to shew why "the world of art *should* be more narrow than the world of nature"—that it should be thereby rendered more monotonous does not follow.

It is well known, that the "world of nature" consists of productions, including objects of every degree of beauty from the very lowest to the highest, and calculated to suit not only the tastes arising from various degrees of intellect, but those arising from the natural instincts of the lower animals. On the other hand, "the world of art," being devoted to the gratification and improvement of intelligent minds only, is therefore narrowed in its scope by the exclusion from its productions of the lower degrees of beauty—even mediocrity is inadmissible; and we know that the science of the ancient Greek artists enabled them to excel the highest individual productions of nature in the perfection of symmetrical beauty. Consequently, all objects in nature are not equally well adapted for artistic study, and it therefore requires, on the part of the artist, besides true genius, much experience and care to enable him to choose proper subjects from nature; and it is in the choice of such subjects, and not in plagiarism from the ancients, that he should select with knowledge and adapt with wisdom. Hence, all such latitudinarian doctrines as those I have quoted must act as a check upon the progress of knowledge in the scientific truth of art. I have observed in some of my works, that in this country a course had been followed in our search for the true science of beauty not differing from that by which the alchymists of the middle ages conducted their investigations; for our ideas of visible beauty are still undefined, and our attempts to produce it in the various branches of art are left dependant, in a great measure, upon chance. Our schools are conducted without reference to any first principles or definite laws of beauty, and from the drawing of a simple architectural moulding to the intricate combinations of form in the human figure, the pupils trust to their hands and eyes alone, servilely and mechanically copying the works of the ancients, instead of being instructed in the unerring principles of science, upon which the beauty of those works normally depends. The instruction they receive is imparted without reference to the judgment or understanding, and they are thereby led to imitate effects without investigating causes. Doubtless, men of great genius sometimes arrive at excellence in the arts of design without a knowledge of the principles upon which beauty of form is based; but it should be kept in mind, that true genius includes an intuitive perception of those principles along with its creative power. It is, therefore, to the generality of mankind that instruction in the definite laws of beauty will be of most service, not only in improving the practice of those who follow the arts professionally, but in enabling all of us to distinguish the true from the false, and to exercise a sound and discriminating taste in forming our judgment upon artistic productions. Æsthetic culture should consequently supersede servile copying, as the basis of instruction in our schools of art. Many teachers of drawing, however, still assert, that, by copying the great works of the ancients, the mind of the pupil will become imbued with ideas similar to theirs—that he will imbibe their feeling for the beautiful, and thereby become inspired with their genius, and think as they thought. To study carefully and to investigate the principles which constitute the excellence of the works of the ancients, is no doubt of much benefit to the student; but it would be as unreasonable to suppose that he should become inspired with artistic genius by merely copying them, as it would be to imagine, that, in literature, poetic inspiration could be created by making boys transcribe or repeat the works of the ancient poets. Sir Joshua Reynolds considered copying as a delusive kind of industry, and has observed, that "Nature herself is not to be too closely copied," asserting that "there are excellences in the art of painting beyond what is commonly called the imitation of nature," and that "a mere copier of nature can never produce any thing great." Proclus, an eminent philosopher and mathematician of the later Platonist school (A.D. 485), says, that "he who takes for his model such forms as nature produces, and confines himself to an exact imitation of these, will never attain to what is perfectly beautiful. For the works of nature are full of disproportion, and fall very short of the true standard of beauty."

It is remarked by Mr J. C. Daniel, in the introduction to his translation of M. Victor Cousin's "Philosophy of the Beautiful," that "the English writers have advocated no theory which allows the beautiful to be universal and absolute; nor have they professedly founded their views on original and ultimate principles. Thus the doctrine of the English school has for the most part been, that beauty is mutable and special, and the inference that has been drawn from this teaching is, that all tastes are equally just, provided that each man speaks of what he feels." He then observes, that the German, and some of the French writers, have thought far differently; for with them the beautiful is "simple, immutable, absolute, though its *forms* are manifold."

So far back as the year 1725, the same truths advanced by the modern German and French writers, and so eloquently illustrated by M. Cousin, were given to the world by Hutchison in his "Inquiry into the Original of our Ideas of Beauty and Virtue." This author says—"We, by absolute beauty, understand only that beauty which we perceive in objects, without comparison to any thing external, of which the object is supposed an imitation or picture, such as the beauty perceived from the works of nature, artificial forms, figures, theorems. Comparative or relative beauty is that which we perceive in objects commonly considered as imitations or resemblances of something else."

Dr Reid also, in his "Intellectual Powers of Man," says—"That taste, which we may call rational, is that part of our constitution by which we are made to receive pleasure from the contemplation of what we conceive to be excellent in its kind, the pleasure being annexed to this judgment, and regulated by it. This taste may be true or false, according as it is founded on a true or false judgment. And if it may be true or false, it must have first principles."

M. Victor Cousin's opinion upon this subject is, however, still more conclusive. He observes—"If the idea of the beautiful is not absolute, like the idea of the true—if it is nothing more than the expression of individual sentiment, the rebound of a changing sensation, or the result of each person's fancy—then the discussions on the fine arts waver without support, and will never end. For a theory of the fine arts to be possible, there must be something absolute in beauty, just as there must be something absolute in the idea of goodness, to render morals a possible science."

The basis of the science of beauty must thus be founded upon fixed principles, and when these principles are evolved with the same care which has characterised the labours of investigators in natural science, and are applied in the fine arts as the natural sciences have been in the useful arts, a solid foundation will be laid, not only for correct practice, but also for a just appreciation of productions in every branch of the arts of design.

We know that the mind receives pleasure through the sense of hearing, not only from the music of nature, but from the euphony of prosaic composition, the rhythm of poetic measure, the artistic composition of successive harmony in simple melody, and the combined harmony of counterpoint in the more complex works of that art. We know, also, that the mind is similarly gratified through the sense of seeing, not only by the visible beauties of nature, but by those of art, whether in symmetrical or picturesque compositions of forms, or in harmonious arrangements of gay or sombre colouring.

Now, in respect to the first of these modes of sensation, we know, that from the time of Pythagoras, the fact has been established, that in whatever manner nature or art may address the ear, the degree of obedience paid to the fundamental law of harmony will determine the presence and degree of that beauty with which a perfect organ can impress a well-constituted mind; and it is my object in this, as it has been in former attempts, to prove it consistent with scientific truth, that that beauty which is addressed to the mind by objects of nature and art, through the eye, is similarly governed. In short, to shew that, as in compositions of sounds, there can be no true beauty in the absence of a strict obedience to this great law of nature, neither can there exist, in compositions of forms or colours, that principle of unity in variety which constitutes beauty, unless such compositions are governed by the same law.

Although in the songs of birds, the gurgling of brooks, the sighing of the gentle summer winds, and all the other beautiful music of nature, no analysis might be able to detect the operation of any precise system of harmony, yet the pleasure thus afforded to the human mind we know to arise from its responding to every development of an obedience to this law. When, in like manner, we find even in those compositions of forms and colours which constitute the wildest and most rugged of Nature's scenery, a species of picturesque grandeur and beauty to which the mind as readily responds as to her more mild and pleasing aspects, or to her sweetest music, we may rest assured that this beauty is simply another development of, and response to, the same harmonic law, although the precise nature of its operation may be too subtle to be easily detected.

The *resumé* of the various works I have already published upon the subject, along with the additional illustrations I am about to lay before my readers, will, I trust, point out a system of harmony, which, in formative art, as well as in that of colouring, will rise superior to the idiosyncracies of different artists, and bring back to one common type the sensations of the eye and the ear, thereby improving that knowledge of the laws of the universe which it is as much the business of science to combine with the ornamental as with the useful arts.

In attempting this, however, I beg it may be understood, that I do not believe any system, based even upon the laws of nature, capable of forming a royal road to the perfection of art, or of "mapping the mighty maze of a creative mind." At the same time, however, I must continue to reiterate the fact, that the diffusion of a general knowledge of the science of visible beauty will afford latent artistic genius just such a vantage ground as that which the general knowledge of philology diffused throughout this country affords its latent literary genius. Although *mere learning* and *true genius* differ as much in the practice of art as they do in the practice of literature, yet a precise and systematic education in the true science of beauty must certainly be as useful in promoting the practice and appreciation of the one, as a precise and systematic education in the science of philology is in promoting the practice and appreciation of the other.

As all beauty is the result of harmony, it will be requisite here to remark, that harmony is not a simple quality, but, as Aristotle defines it, "the union of contrary principles having a ratio to each other." Harmony thus operates in the production of all that is beautiful in nature, whether in the combinations, in the motions, or in the affinities of the elements of matter.

The contrary principles to which Aristotle alludes, are those of uniformity and variety; for, according to the predominance of the one or the other of these principles, every kind of beauty is characterised. Hence the difference between symmetrical and picturesque beauty:—the first allied to the principle of uniformity, in being based upon precise laws that may be taught so as to enable men of ordinary capacity to produce it in their works—the second allied to the principle of variety often to so great a degree that they yield an obedience to the precise principles of harmony so subtilely, that they cannot be detected in its constitution, but are only felt in the response by which true genius acknowledges their presence. The generality of mankind may be capable of perceiving this latter kind of beauty, and of feeling its effects upon the mind, but men of genius, only, can impart it to works of art, whether addressed to the eye or the ear. Throughout the sounds, forms, and colours of nature, these two kinds of beauty are found not only in distinct developments, but in every degree of amalgamation. We find in the songs of some birds, such as those of the chaffinch, thrush, &c., a rhythmical division, resembling in some measure the symmetrically precise arrangements of parts which characterises all artistic musical composition; while in the songs of other birds, and in the other numerous melodies with which nature charms and soothes the mind, there is no distinct regularity in the division of their parts. In the forms of nature, too, we find amongst the innumerable flowers with which the surface of the earth is so profusely decorated, an almost endless variety of systematic arrangements of beautiful figures, often so perfectly symmetrical in their combination, that the most careful application of the angleometer could scarcely detect the slightest deviation from geometrical precision; while, amongst the masses of foliage by which the forms of many trees are divided and subdivided into parts, as also amongst the hills and valleys, the mountains and ravines, which divide the earth's surface, we find in every possible variety of aspect the beauty produced by that irregular species of symmetry which characterises the picturesque.

In like manner, we find in wild as well as cultivated flowers the most symmetrical distributions of colours accompanying an equally precise species of harmony in their various kinds of contrasts, often as mathematically regular as the geometric diagrams by which writers upon colour sometimes illustrate their works; while in the general colouring of the picturesque beauties of nature, there is an endless variety in its distributions, its blendings, and its modifications. In the forms and colouring of animals, too, the same endless variety of regular and irregular symmetry is to be found. But the highest degree of beauty in nature is the result of an equal balance of uniformity with variety. Of this the human figure is an example; because, when it is of those proportions universally acknowledged to be the most perfect, its uniformity bears to its variety an apparently equal ratio. The harmony of combination in the normal proportions of its parts, and the beautifully simple harmony of succession in the normal melody of its softly undulating outline, are the perfection of symmetrical beauty, while the innumerable changes upon the contour which arise from the actions and attitudes occasioned by the various emotions of the mind, are calculated to produce every species of picturesque beauty, from the softest and most pleasing to the grandest and most sublime.

Amongst the purely picturesque objects of inanimate nature, I may, as in a former work, instance an ancient oak tree, for its beauty is enhanced by want of apparent symmetry. Thus, the more fantastically crooked its branches, and the greater the dissimilarity and variety it exhibits in its masses of foliage, the more beautiful it appears to the artist and the amateur; and, as in the human figure, any attempt to produce variety in the proportions of its lateral halves would be destructive of its symmetrical beauty, so in the oak tree any attempt to produce palpable similarity between any of its opposite sides would equally deteriorate its picturesque beauty. But picturesque beauty is not the result of the total absence of symmetry; for, as none of the irregularly constructed music of nature could be pleasing to the ear unless there existed in the arrangement of its notes an obedience, however subtle, to the great harmonic law of Nature, so neither could any object be picturesquely beautiful, unless the arrangement of its parts yields, although it may be obscurely, an obedience to the same law.

However symmetrically beautiful any architectural structure may be, when in a complete and perfect state, it must, as it proceeds towards ruin, blend the picturesque with the symmetrical; but the type of its beauty will continue to be the latter, so long as a sufficient portion of it remains to convey an idea of its original perfection. It is the same with the human form and countenance; for age does not destroy their original beauty, but in both only lessens that which is symmetrical, while it increases that which is picturesque.

In short, as a variety of simultaneously produced sounds, which do not relate to each other agreeably to this law, can only convey to the mind a feeling of mere noise; so a variety of forms or colours simultaneously exposed to the eye under similar circumstances, can only convey to the mind a feeling of chaotic confusion, or what may be termed *visible* discord. As, therefore, the two principles of uniformity and variety, or similarity and dissimilarity, are in operation in every harmonious combination of the elements of sound, of form, and of colour, we must first have recourse to numbers in the abstract before we can form a proper basis for a universal science of beauty.

THE SCIENCE OF BEAUTY EVOLVED FROM THE HARMONIC LAW OF NATURE, AGREEABLY TO THE PYTHAGOREAN SYSTEM OF NUMERICAL RATIO.

The scientific principles of beauty appear to have been well known to the ancient Greeks; and it must have been by the practical application of that knowledge to the arts of Design, that that people continued for a period of upwards of three hundred years to execute, in every department of these arts, works surpassing in chaste beauty any that had ever before appeared, and which have not been equalled during the two thousand years which have since elapsed.

Æsthetic science, as the science of beauty is now termed, is based upon that great harmonic law of nature which pervades and governs the universe. It is in its nature neither absolutely physical nor absolutely metaphysical, but of an intermediate nature, assimilating in various degrees, more or less, to one or other of those opposite kinds of science. It specially embodies the inherent principles which govern impressions made upon the mind through the senses of hearing and seeing. Thus, the æsthetic pleasure derived from listening to the beautiful in musical composition, and from contemplating the beautiful in works of formative art, is in both cases simply a response in the human mind to artistic developments of the great harmonic law upon which the science is based.

Although the eye and the ear are two different senses, and, consequently, various in their modes of receiving impressions; yet the sensorium is but one, and the mind by which these impressions are perceived and appreciated is also characterised by unity. There appears, likewise, a striking analogy between the natural constitution of the two kinds of beauty, which is this, that the more physically æsthetic elements of the highest works of musical composition are melody, harmony, and tone, whilst those of the highest works of formative art are contour, proportion, and colour. The melody or theme of a musical composition and its harmony are respectively analogous,—1st, To the outline of an artistic work of formative art; and 2d, To the proportion which exists amongst its parts. To the careful investigator these analogies become identities in their effect upon the mind, like those of the more metaphysically æsthetic emotions produced by expression in either of these arts.

Agreeably to the first analogy, the outline and contour of an object, suppose that of a building in shade when viewed against a light background, has a similar effect upon the mind with that of the simple melody of a musical composition when addressed to the ear unaccompanied by the combined harmony of counterpoint. Agreeably to the second analogy, the various parts into which the surface of the supposed elevation is divided being simultaneously presented to the eye, will, if arranged agreeably to the same great law, affect the mind like that of an equally harmonious arrangement of musical notes accompanying the supposed melody.

There is, however, a difference between the construction of these two organs of sense, viz., that the ear must in a great degree receive its impressions involuntarily; while the eye, on the other hand, is provided by nature with the power of either dwelling upon, or instantly shutting out or withdrawing itself from an object. The impression of a sound, whether simple or complex, when made upon the ear, is instantaneously conveyed to the mind; but when the sound ceases, the power of observation also ceases. But the eye can dwell upon objects presented to it so long as they are allowed to remain pictured on the retina; and the mind has thereby the power of leisurely examining and comparing them. Hence the ear guides more as a mere sense, at once and without reflection; whilst the eye, receiving its impressions gradually, and part by part, is more directly under the influence of mental analysis, consequently producing a more metaphysically æsthetic emotion. Hence, also, the acquired power of the mind in appreciating impressions made upon it through the organ of sight under circumstances, such as perspective, &c., which to those who take a hasty view of the subject appear impossible.

Dealing as this science therefore does, alike with the sources and the resulting principles of beauty, it is scarcely less dependent on the accuracy of the senses than on the power of the understanding, inasmuch as the effect which it produces is as essential a property of objects, as are its laws inherent in the human mind. It necessarily comprehends a knowledge of those first principles in art, by which certain combinations of sounds, forms, and colours produce an effect upon the mind, connected, in the first instance, with sensation, and in the second with the reasoning faculty. It is, therefore, not only the basis of all true practice in art, but of all sound judgment on questions of artistic criticism, and necessarily includes those laws whereon a correct taste must be based.

Doubtless many eloquent and ingenious treatises have been written upon beauty and taste ; but in nearly every case, with no other effect than that of involving the subject in still greater uncertainty. Even when restricted to the arts of design, they have failed to exhibit any definite principles whereby the true may be distinguished from the false, and some natural and recognised laws of beauty reduced to demonstration. This may be attributed, in a great degree, to the neglect of a just discrimination between what is merely agreeable, or capable of exciting pleasurable sensations, and what is essentially beautiful; but still more to the confounding of the operations of the understanding with those of the imagination. Very slight reflection, however, will suffice to shew how essentially distinct these two faculties of the mind are; the former being regulated, in matters of taste, by irrefragable principles existing in nature, and responded to by an inherent principle existing in the human mind; while the latter operates in the production of ideal combinations of its own creation, altogether independent of any immediate impression made upon the senses. The beauty of a flower, for example, or of a dew-drop, depends on certain combinations of form and colour, manifestly referable to definite and systematic, though it may be unrecognised, laws; but when Oberon, in "Midsummer Night's Dream," is made to exclaim—

> "And that same dew, which sometimes on the buds
> Was wont to swell, like round and orient pearls,
> Stood now within the pretty floweret's eyes,
> Like tears that did their own disgrace bewail,"—

the poet introduces a new element of beauty equally legitimate, yet altogether distinct from, although accompanying that which constitutes the more precise science of æsthetics as here defined. The composition of the rhythm is an operation of the understanding, but the beauty of the poetic fancy is an operation of the imagination.

Our physical and mental powers, æsthetically considered, may therefore be classed under three heads, in their relation to the fine arts, viz., the receptive, the perceptive, and the conceptive.

The senses of hearing and seeing are respectively, in the degree of their physical power, receptive of impressions made upon them, and of these impressions the sensorium, in the degree of its mental power, is perceptive. This perception enables the mind to form a judgment whereby it appreciates the nature and quality of the impression originally made on the receptive organ. The mode of this operation is intuitive, and the quickness and accuracy with which the nature and quality of the impression is apprehended, will be in the degree of the intellectual vigour of the mind by which it is perceived. Thus we are, by the cultivation of these intuitive faculties, enabled to decide with accuracy as to harmony or discord, proportion or deformity, and assign sound reasons for our judgment in matters of taste. But mental conception is the intuitive power of constructing original ideas from these materials; for after the receptive power has acted, the perception operates in establishing facts, and then the judgment is formed upon these operations by the reasoning powers, which lead, in their turn, to the creations of the imagination.

The power of forming these creations is the true characteristic of genius, and determines the point at which art is placed beyond all determinable canons,—at which, indeed, æsthetics give place to metaphysics.

In the science of beauty, therefore, the human mind is the subject, and the effect of external nature, as well as of works of art, the object. The external world, and the individual mind, with all that lies within the scope of its powers, may be considered as two separate existences, having a distinct relation to each other. The subject is affected by the object, through that inherent faculty by which it is enabled to respond to every development of the all-governing harmonic law of nature; and the media of communication are the senorium and its inlets—the organs of sense.

This harmonic law of nature was either originally discovered by that illustrious philosopher Pythagoras, upwards of five hundred years before Christ, or a knowledge of it obtained by him about that period, from the Egyptian or Chaldean priests. For after having been initiated into all the Grecian and barbarian sacred mysteries, he went to Egypt, where he remained upwards of twenty years, studying in the colleges of its priests; and from Egypt he went into the East, and visited the Persian and Chaldean magi.*

By the generality of the biographers of Pythagoras, it is said to be difficult to give a clear idea of his philosophy, as it is almost certain he never committed it to writing, and that it has been disfigured by the fantastic dreams and chimeras of later Pythagoreans. Diogenes Laërtius, however, whose "Lives of the Philosophers" was supposed to be written about the end of the second century of our era, says "there are three volumes extant written by Pythagoras. One on education, one on politics, and one on natural philosophy." And adds, that there were several other books extant, attributed to Pythagoras, but which were not written by him. Also, in his "Life of Philolaus," that Plato wrote to Dion to take care and purchase the books of Pythagoras.† But whether this great

* Diogenes Laërtius's "Lives of the Philosophers," literally translated. Bohn: London.

† Ibid.

philosopher committed his discoveries to writing or not, his doctrines regarding the philosophy of beauty are well-known to be, that he considered numbers as the essence and the principle of all things, and attributed to them a real and distinct existence; so that, in his view, they were the elements out of which the universe was constructed, and to which it owed its beauty. Diogenes Laërtius gives the following account of this law:—"That the monad was the beginning of everything. From the monad proceeds an indefinite duad, which is subordinate to the monad as to its cause. That from the monad and indefinite duad proceeds numbers. That the part of science to which Pythagoras applied himself above all others, was arithmetic; and that he taught 'that from numbers proceed signs, and from these latter, lines, of which plane figures consist; that from plane figures are derived solid bodies; that of all plane figures the most beautiful was the circle, and of all solid bodies the most beautiful was the sphere.' He discovered the numerical relations of sounds on a single string; and taught that everything owes its existence and consistency to harmony. In so far as I know, the most condensed account of all that is known of the Pythagorian system of numbers is the following:—'The monad or unity is that quantity, which, being deprived of all number, remains fixed. It is the fountain of all number. The duad is imperfect and passive, and the cause of increase and division. The triad, composed of the monad and duad, partakes of the nature of both. The tetrad, tetractys, or quaternion number is most perfect. The decad, which is the sum of the four former, comprehends all arithmetical and musical proportions.'" *

These short quotations, I believe, comprise all that is known,

* Rose's "Biographical Dictionary."

for certain, of the manner in which Pythagoras systematised the law of numbers. Yet, from the teachings of this great philosopher and his disciples, the harmonic law of nature, in which the fundamental principles of beauty are embodied, became so generally understood and universally applied in practice throughout all Greece, that the fragments of their works, which have reached us through a period of two thousand years, are still held to be examples of the highest artistic excellence ever attained by mankind. In the present state of art, therefore, a knowledge of this law, and of the manner in which it may again be applied in the production of beauty in all works of form and colour, must be of singular advantage; and the object of this work is to assist in the attainment of such a knowledge.

It has been remarked, with equal comprehensiveness and truth, by a writer* in the *British and Foreign Medical Review*, that "there is harmony of numbers in all nature—in the force of gravity—in the planetary movements—in the laws of heat, light, electricity, and chemical affinity—in the forms of animals and plants—in the perceptions of the mind. The direction, indeed, of modern natural and physical science is towards a generalization which shall express the fundamental laws of all by one simple numerical ratio. And we think modern science will soon shew that the mysticism of Pythagoras was mystical only to the unlettered, and that it was a system of philosophy founded on the then existing mathematics, which latter seem to have comprised more of the philosophy of numbers than our present." Many years of careful investigation have convinced me of the truth of this remark, and of the great advantage derivable from an application of the Pythagorean system in the arts of design. For so

* Professor Laycock, now of the University of Edinburgh.

simple is its nature, that any one of an ordinary capacity of mind, and having a knowledge of the most simple rules of arithmetic, may, in a very short period, easily comprehend its nature, and be able to apply it in practice.

The elements of the Pythagorean system of harmonic number, so far as can be gathered from the quotations I have given above, seem to be simply the indivisible monad (1); the duad (2), arising from the union of one monad with another; the triad (3), arising from the union of the monad with the duad; and the tetrad (4), arising from the union of one duad with another, which tetrad is considered a perfect number. From the union of these four elements arises the decad (10), the number, which, agreeably to the Pythagorean system, comprehends all arithmetical and harmonic proportions. If, therefore, we take these elements and unite them progressively in the following order, we shall find the series of harmonic numbers (2), (3), (5), and (7), which, with their multiples, are the complete numerical elements of all harmony, thus:—

$$1 + 1 = 2$$
$$1 + 2 = 3$$
$$2 + 3 = 5$$
$$3 + 4 = 7$$

In order to render an extended series of harmonic numbers useful, it must be divided into scales; and it is a rule in the formation of these scales, that the first must begin with the monad (1) and end with the duad (2), the second begin with the duad (2) and end with the tetrad (4), and that the beginning and end of all other scales must be continued in the same arithmetical progression. These primary elements will then form the foundation of a series of such scales.

I.	(1)								(2)
II.	(2)				(3)				(4)
III.	(4)		(5)		(6)		(7)		(8)
IV.	(8)	(9)	(10)	( )	(12)	( )	(14)	(15)	(16)

The first of these scales has in (1) and (2) a beginning and an end; but the second has in (2), (3), and (4) the essential requisites demanded by Aristotle in every composition, viz., "a beginning, a middle, and an end;" while the third has not only these essential requisites, but two intermediate parts (5) and (7), by which the beginning, the middle, and the end are united. In the fourth scale, however, the arithmetical progression is interrupted by the omission of numbers 11 and 13, which, not being multiples of either (2), (3), (5), or (7), are inadmissible.

Such is the nature of the harmonic law which governs the progressive scales of numbers by the simple multiplication of the monad.

I shall now use these numbers as divisors in the formation of a series of four such scales of parts, which has for its primary element, instead of the indivisible monad, a quantity which may be indefinitely divided, but which cannot be added to or multiplied. Like the monad, however, this quantity is represented by (1). The following is this series of four scales of harmonic parts:—

I.	(1)								($\frac{1}{2}$)
II.	($\frac{1}{2}$)				($\frac{1}{3}$)				($\frac{1}{4}$)
III.	($\frac{1}{4}$)		($\frac{1}{5}$)		($\frac{1}{6}$)		($\frac{1}{7}$)		($\frac{1}{8}$)
IV.	($\frac{1}{8}$)	($\frac{1}{9}$)	($\frac{1}{10}$)	( )	($\frac{1}{12}$)	( )	($\frac{1}{14}$)	($\frac{1}{15}$)	($\frac{1}{16}$)

The scales I., II., and III. may now be rendered as complete as scale IV., simply by multiplying upwards by 2 from $(\frac{1}{9})$, $(\frac{1}{5})$, $(\frac{1}{3})$, $(\frac{1}{7})$, and $(\frac{1}{15})$, thus:—

I.	(1)	$(\frac{8}{9})$	$(\frac{4}{5})$		$(\frac{2}{3})$		$(\frac{4}{7})$	$(\frac{8}{15})$	$(\frac{1}{2})$
II.	$(\frac{1}{2})$	$(\frac{4}{9})$	$(\frac{2}{5})$		$(\frac{1}{3})$		$(\frac{2}{7})$	$(\frac{4}{15})$	$(\frac{1}{4})$
III.	$(\frac{1}{4})$	$(\frac{2}{9})$	$(\frac{1}{5})$		$(\frac{1}{6})$		$(\frac{1}{7})$	$(\frac{2}{15})$	$(\frac{1}{8})$
IV.	$(\frac{1}{8})$	$(\frac{1}{9})$	$(\frac{1}{10})$	( )	$(\frac{1}{12})$	( )	$(\frac{1}{14})$	$(\frac{1}{15})$	$(\frac{1}{16})$

We now find between the beginning and the end of scale I. the quantities $(\frac{8}{9})$, $(\frac{4}{5})$, $(\frac{2}{3})$, $(\frac{4}{7})$, and $(\frac{8}{15})$.

The three first of these quantities we find to be the remainders of the whole indefinite quantity contained in (1), after subtracting from it the primary harmonic quantities $(\frac{1}{9})$, $(\frac{1}{5})$, and $(\frac{1}{3})$; we, however, find also amongst these harmonic quantities that of $(\frac{1}{4})$, which being subtracted from (1) leaves $(\frac{3}{4})$, a quantity the most suitable whereby to fill up the hiatus between $(\frac{4}{5})$ and $(\frac{2}{3})$ in scale I., which arises from the omission of $(\frac{1}{11})$ in scale IV. In like manner we find the two last of these quantities, $(\frac{4}{7})$ and $(\frac{8}{15})$, are respectively the largest of the two parts into which 7 and 15 are susceptible of being divided. Finding the number 5 to be divisible into parts more unequal than (2) to (3) and less unequal than (4) to (7), $(\frac{3}{5})$ naturally fills up the hiatus between these quantities in scale I., which hiatus arises from the omission of $(\frac{1}{13})$ in scale IV. Thus:—

I.	(1)	$(\frac{8}{9})$	$(\frac{4}{5})$	$(\frac{3}{4})$	$(\frac{2}{3})$	$(\frac{3}{5})$	$(\frac{4}{7})$	$(\frac{8}{15})$	$(\frac{1}{2})$
II.	$(\frac{1}{2})$	$(\frac{4}{9})$	$(\frac{2}{5})$	( )	$(\frac{1}{3})$	( )	$(\frac{2}{7})$	$(\frac{4}{15})$	$(\frac{1}{4})$
III.	$(\frac{1}{4})$	$(\frac{2}{9})$	$(\frac{1}{5})$	( )	$(\frac{1}{6})$	( )	$(\frac{1}{7})$	$(\frac{2}{15})$	$(\frac{1}{8})$
IV.	$(\frac{1}{8})$	$(\frac{1}{9})$	$(\frac{1}{10})$	( )	$(\frac{1}{12})$	( )	$(\frac{1}{14})$	$(\frac{1}{15})$	$(\frac{1}{16})$

Scale I. being now complete, we have only to divide these latter quantities by (2) downwards in order to complete the other three. Thus:—

I.	(1)	$(\frac{8}{9})$	$(\frac{4}{5})$	$(\frac{3}{4})$	$(\frac{2}{3})$	$(\frac{3}{5})$	$(\frac{4}{7})$	$(\frac{8}{15})$	$(\frac{1}{2})$
II.	$(\frac{1}{2})$	$(\frac{4}{9})$	$(\frac{2}{5})$	$(\frac{3}{8})$	$(\frac{1}{3})$	$(\frac{3}{10})$	$(\frac{2}{7})$	$(\frac{4}{15})$	$(\frac{1}{4})$
III.	$(\frac{1}{4})$	$(\frac{2}{9})$	$(\frac{1}{5})$	$(\frac{3}{16})$	$(\frac{1}{6})$	$(\frac{3}{20})$	$(\frac{1}{7})$	$(\frac{2}{15})$	$(\frac{1}{8})$
IV.	$(\frac{1}{8})$	$(\frac{1}{9})$	$(\frac{1}{10})$	$(\frac{3}{32})$	$(\frac{1}{12})$	$(\frac{3}{40})$	$(\frac{1}{14})$	$(\frac{1}{15})$	$(\frac{1}{16})$

The harmony existing amongst these numbers or quantities consists of the numerical relations which the parts bear to the whole and to each other; and the more simple these relations are, the more perfect is the harmony. The following are the numerical harmonic ratios which the parts bear to the whole:—

I.	(1 : 1)	(8 : 9)	(4 : 5)	(3 : 4)	(2 : 3)	(3 : 5)	(4 : 7)	(8 : 15)	(1 : 2)
II.	(1 : 2)	(4 : 9)	(2 : 5)	(3 : 8)	(1 : 3)	(3 : 10)	(2 : 7)	(4 : 15)	(1 : 4)
III.	(1 : 4)	(2 : 9)	(1 : 5)	(3 : 16)	(1 : 6)	(3 : 20)	(1 : 7)	(2 : 15)	(1 : 8)
IV.	(1 : 8)	(1 : 9)	(1 : 10)	(3 : 32)	(1 : 12)	(3 : 40)	(1 : 14)	(1 : 15)	(1 : 16)

The following are the principal numerical relations which the parts in each scale bear to one another:—

$$(\tfrac{1}{2}) : (\tfrac{4}{7}) = (7 : 8)$$
$$(\tfrac{4}{5}) : (\tfrac{8}{9}) = (9 : 10)$$
$$(\tfrac{2}{3}) : (\tfrac{4}{5}) = (5 : 6)$$
$$(\tfrac{4}{7}) : (\tfrac{2}{3}) = (6 : 7)$$
$$(\tfrac{8}{15}) : (\tfrac{4}{7}) = (14 : 15)$$
$$(\tfrac{1}{2}) : (\tfrac{8}{15}) = (15 : 16)$$

Although these relations are exemplified by parts of scale I., the same ratios exist between the relative parts of scales II., III., and IV., and would exist between the parts of any other scales that might be added to that series.

These are the simple elements of the science of that harmony which pervades the universe, and by which the various kinds of beauty æsthetically impressed upon the senses of hearing and seeing are governed.

THE SCIENCE OF BEAUTY AS APPLIED TO SOUNDS.

It is well-known that all sounds arise from a peculiar action of the air, and that this action may be excited by the concussion resulting from the sudden displacement of a portion of the atmosphere itself, or by the rapid motions of bodies, or of confined columns of air ; in all which cases, when the motions are irregular, and the force great, the sound conveyed to the sensorium is called a noise. But that musical sounds are the result of equal and regular vibratory motions, either of an elastic body, or of a column of air in a tube, exciting in the surrounding atmosphere a regular and equal pulsation. The ear is the medium of communication between those varieties of atmospheric action and the seat of consciousness. To describe fully the beautiful arrangement of the various parts of this organ, and their adaptation to the purpose of collecting and conveying these undulatory motions of the atmosphere, is as much beyond the scope of my present attempt as it is beyond my anatomical knowledge; but I may simply remark, that within the ear, and most carefully protected in the construction of that organ, there is a small cavity containing a pellucid fluid, in which the minute extremities of the auditory nerve float; and that this fluid is the last of the media through which the action producing the sensation of sound is conveyed to the nerve, and thence to the sensorium, where its nature becomes perceptible to the mind.

The impulses which produce musical notes must arrive at a certain frequency before the ear loses the intervals of silence between them, and is impressed by only one continued sound; and as they increase in frequency the sound becomes more acute upon the ear. The pitch of a musical note is, therefore, determined by the frequency, of these impulses; but, on the other hand, its intensity or loudness will depend upon the violence and the quality of its tone on the material employed in producing them. All such sounds, therefore, whatever be their loudness or the quality of their tone in which the impulses occur with the same frequency are in perfect unison, having the same pitch. Upon this the whole doctrine of harmonies is founded, and by this the laws of numerical ratio are found to operate in the production of harmony, and the theory of music rendered susceptible of exact reasoning.

The mechanical means by which such sounds can be produced are extremely various; but, as it is my purpose simply to shew the nature of harmony of sound as related to, or as evolving numerical harmonic ratio, I shall confine myself to the most simple mode of illustration—namely, that of the monochord. This is an instrument consisting of a string of a given length stretched between two bridges standing upon a graduated scale. Suppose this string to be stretched until its tension is such that, when drawn a little to a side and suddenly let go, it would vibrate at the rate of 64 vibrations in a second of time, producing to a certain distance in the surrounding atmosphere a series of pulsations of the same frequency.

These pulsations will communicate through the ear a musical note which would, therefore, be the fundamental note of such a string. Now, the phenomenon said to be discovered by Pythagoras is well known to those acquainted with the science of acoustics, namely, that immediately after the string is thus put into vibratory motion, it spontaneously divides itself, by a node, into two equal parts, the vibrations of each of which occur with a double frequency—namely, 128 in a second of time, and, consequently, produce a note doubly acute in pitch, although much weaker as to intensity or loudness; that it then, while performing these two series of vibrations, divides itself, by two nodes, into three parts, each of which vibrates with a frequency triple that of the whole string; that is, performs 192 vibrations in a second of time, and produces a note corresponding in increase of acuteness, but still less intense than the former, and that this continues to take place in the arithmetical progression of 2, 3, 4, &c. Simultaneous vibrations, agreeably to the same law of progression, which, however, seem to admit of no other primes than the numbers 2, 3, 5, and 7, are easily excited upon any stringed instrument, even by the lightest possible touch of any of its strings while in a state of vibratory motion, and the notes thus produced are distinguished by the name of harmonics. It follows, then, that one-half of a musical string, when divided from the whole by the pressure of the finger, or any other means, and put into vibratory motion, produces a note doubly acute to that produced by the vibratory motion of the whole string; the third part, similarly separated, a note trebly acute; and the same with every part into which any musical string may be divided. This is the fundamental principle by which all stringed instruments are made to produce harmony. It is the same with wind instruments, the sounds of which are produced by the frequency of the pulsations occasioned in the surrounding atmosphere by agitating a column of air confined within a tube as in an organ, in which the frequency of pulsation becomes greater in an inverse ratio to the length of the pipes. But the following series of four successive scales of musical notes will give the reader a more comprehensive view of the manner in which they follow the law of numerical ratio just explained than any more lengthened exposition.

It is here requisite to mention, that in the construction of these scales, I have not only adopted the old German or literal mode of indicating the notes, but have included, as the Germans do, the note termed by us B flat as B natural, and the note we term B natural as H. Now, although this arrangement differs from that followed in the construction of our modern Diatonic scale, yet as the ratio of 4 : 7 is more closely related to that of 1 : 2 than that of 8 : 15, and as it is offered by nature in the spontaneous division of the monochord, I considered it quite admissible. The figures give the parts of the monochord which would produce the notes.

I.	(1)	$(\frac{8}{9})$	$(\frac{4}{5})$	$(\frac{3}{4})$	$(\frac{2}{3})$	$(\frac{3}{5})$	$(\frac{4}{7})$	$(\frac{8}{15})$	$(\frac{1}{2})$*
	C	D	E	F	G	A	B	H	*c*
II.	$(\frac{1}{2})$*	$(\frac{4}{9})$	$(\frac{2}{5})$	$(\frac{3}{8})$	$(\frac{1}{3})$*	$(\frac{3}{10})$	$(\frac{2}{7})$	$(\frac{2}{15})$	$(\frac{1}{4})$*
	c	*d*	*e*	*f*	*g*	*a*	*b*	*h*	$\bar{c}$
III.	$(\frac{1}{4})$*	$(\frac{2}{9})$	$(\frac{1}{5})$*	$(\frac{3}{16})$	$(\frac{1}{6})$*	$(\frac{3}{20})$	$(\frac{1}{7})$*	$(\frac{2}{15})$	$(\frac{1}{8})$*
	$\bar{c}$	$\bar{d}$	$\bar{e}$	$\bar{f}$	$\bar{g}$	$\bar{a}$	$\bar{b}$	$\bar{h}$	$\bar{\bar{c}}$
IV.	$(\frac{1}{8})$*	$(\frac{1}{9})$*	$(\frac{1}{10})$*	$(\frac{3}{32})$	$(\frac{1}{12})$*	$(\frac{3}{40})$	$(\frac{1}{14})$*	$(\frac{1}{15})$*	$(\frac{1}{16})$*
	$\bar{\bar{c}}$	$\bar{\bar{d}}$	$\bar{\bar{e}}$	$\bar{\bar{f}}$	$\bar{\bar{g}}$	$\bar{\bar{a}}$	$\bar{\bar{b}}$	$\bar{\bar{h}}$	$\bar{\bar{\bar{c}}}$

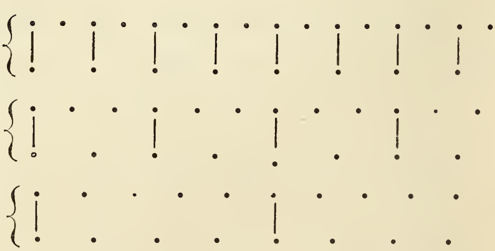
The notes marked (*) are the harmonics which naturally arise from the division of the string by 2, 3, 5, and 7, and the multiples of these primes.

Thus every musical sound is composed of a certain number of parts called pulsations, and these parts must in every scale relate harmonically to some fundamental number. When these parts are multiples of the fundamental number by 2, 4, 8, &c., like the pulsations of the sounds indicated by c, $\bar{c}$, $\bar{\bar{c}}$, $\bar{\bar{\bar{c}}}$, they are called tonic notes, being the most consonant; when the pulsations are similar multiples by 3, 6, 12, &c., like those of the sounds indicated by g, $\bar{g}$, $\bar{\bar{g}}$, they are called dominant notes, being the next most consonant; and multiples by 5, 10, &c., like those of the sounds indicated by e, $\bar{e}$, $\bar{\bar{e}}$, they are called mediant notes, from a similar cause. In harmonic combinations of musical sounds, the æsthetic feeling produced by their agreement depends upon the relations they bear to each other with reference to the number of pulsations produced in a given time by the fundamental note of the scale to which they belong; and it will be observed, that the more simple the numerical ratios are amongst the pulsations of any number of notes simultaneously produced, the more perfect their agreement. Hence the origin of the common chord or fundamental concord in the united sounds of the tonic, the dominant, and the mediant notes, the ratios and coincidences of whose pulsations 2 : 1, 3 : 2, 5 : 4, may thus be exemplified:—

Pulsation of 2 : c / 1 / C

Pulsation of 3 : G / 2 / C

Pulsation of 5 : E / 4 / C



In musical composition, the law of number also governs its division into parts, in order to produce upon the ear, along with the beauty of harmony, that of rhythm. Thus a piece of music is divided into parts each of which contains a certain number of other parts called bars, which may be divided and subdivided into any number of notes, and the performance of each bar is understood to occupy the same portion of time, however numerous the notes it contains may be; so that the music of art is regularly symmetrical in its structure; while that of nature is in general as irregular and indefinite in its rhythm as it is in its harmony.

Thus I have endeavoured briefly to explain the manner in which the law of numerical ratio operates in that species of beauty perceived through the ear.

The definite principles of the art of music founded upon this law have been for ages so systematised that those who are instructed in them advance steadily in proportion to their natural endowments, while those who refuse this instruction rarely attain to any excellence. In the sister arts of form and colour, however, a system of tuition, founded upon this law, is still a desideratum, and a knowledge of the scientific principles by which these arts are governed is confined to a very few, and scarcely acknowledged amongst those whose professions most require their practical application.

THE SCIENCE OF BEAUTY AS APPLIED TO FORMS.

It is justly remarked, in the "Illustrated Record of the New York Exhibition of 1853," that "it is a question worthy of consideration how far the mediocrity of the present day is attributable to an overweening reliance on natural powers and a neglect of the lights of science;" and there is expressed a thorough conviction of the fact that, besides the evils of the copying system, "much genius is now wasted in the acquirement of rudimentary knowledge in the slow school of practical experiment, and that the excellence of the ancient Greek school of design arose from a thoroughly digested canon of form, and the use of geometrical formulas, which make the works even of the second and third-rate genius of that period the wonder and admiration of the present day."

That such a canon of form, and that the use of such geometrical formula, entered into the education, and thereby facilitated the practice of ancient Greek art, I have in a former work expressed my firm belief, which is founded on the remarkable fact, that for a period of nearly three centuries, and throughout a whole country politically divided into states often at war with each other, works of sculpture, architecture, and ornamental design were executed, which surpass in symmetrical beauty any works of the kind produced during the two thousand years that have since elapsed. So decided is this superiority, that the artistic remains of the extraordinary period I alluded to are, in all civilised nations, still held to be the most perfect specimens of formative art in the world; and even when so fragmentary as to be denuded of everything that can convey an idea of expression, they still excite admiration and wonder by the purity of their geometric beauty. And so universal was this excellence, that it seems to have characterised every production of formative art, however humble the use to which it was applied.

The common supposition, that this excellence was the result of an extraordinary amount of genius existing among the Greek people during that particular period, is not consistent with what we know of the progress of mankind in any other direction, and is, in the present state of art, calculated to retard its progress, inasmuch as such an idea would suggest that, instead of making any exertion to arrive at a like general excellence, the world must wait for it until a similar supposed psychological phenomenon shall occur.

But history tends to prove that the long period of universal artistic excellence throughout Greece could only be the result of an early inculcation of some well-digested system of correct elementary principles, by which the ordinary amount of genius allotted to mankind in every age was properly nurtured and cultivated; and by which, also, a correct knowledge and appreciation of art were disseminated amongst the people generally. Indeed, Müller, in his "Ancient Art and its Remains," shews clearly that some certain fixed principles, constituting a science of proportions, were known in Greece, and that they formed the basis of all artists' education and practice during the period referred to; also, that art began to decline, and its brightest period to close, as this science fell into disuse, and the Greek artists, instead of working for an enlightened community, who understood the nature of the principles which guided them, were called upon to gratify the impatient whims of pampered and tyrannical rulers.

By being instructed in this science of proportion, the Greek artists were enabled to impart to their representations of the human figure a mathematically correct species of symmetrical beauty; whether accompanying the slender and delicately undulated form of the Venus,—its opposite, the massive and powerful mould of the Hercules,—or the characteristic representation of any other deity in the heathen mythology. And this seems to have been done with equal ease in the minute figure cut on a precious gem, and in the most colossal statue. The same instruction likewise enabled the architects of Greece to institute those varieties of proportions in structure called the Classical Orders of Architecture; which are so perfect that, since the science which gave them birth has been buried in oblivion, classical architecture has been little more than an imitative art; for all who have since written upon the subject, from Vitruvius downwards, have arrived at nothing, in so far as the great elementary principles in question are concerned, beyond the most vague and unsatisfactory conjectures. For a more clear understanding of the nature of this application of the Pythagorean law of number to the harmony of form, it will be requisite to repeat the fact, that modern science has shewn that the cause of the impression, produced by external nature upon the sensorium, called light, may be traced to a molecular or ethereal action. This action is excited naturally by the sun, artificially by the combustion of various substances, and sometimes physically within the eye. Like the atmospheric pulsations which produce sound, the action which produces light is capable, within a limited sphere, of being reflected from some bodies and transmitted through others; and by this reflection and transmission the visible nature of forms and figures is communicated to the sensorium. The eye is the medium of this communication; and its structural beauty, and perfect adaptation to the purpose of conveying this action, must, like those of the ear, be left to the anatomist fully to describe. It is here only necessary to remark, that the optic nerve, like the auditory nerve, ends in a carefully protected fluid, which is the last of the media interposed between this peculiarly subtle action and the nerve upon which it impresses the presence of the object from which it is reflected or through which it is transmitted, and the nature of such object made perceptible to the mind. The eye and the ear are thus, in one essential point, similar in their physiology, relatively to the means provided for receiving impressions from external nature; it is, therefore, but reasonable to believe that the eye is capable of appreciating the exact subdivision of spaces, just as the ear is capable of appreciating the exact subdivision of intervals of time; so that the division of space into exact numbers of equal parts will æsthetically affect the mind through the medium of the eye.

We assume, therefore, that the standard of symmetry, so estimated, is deduced from the simplest law that could have been conceived—the law that the angles of direction must all bear to some fixed angle the same simple relations which the different notes in a chord of music bear to the fundamental note; that is, relations expressed arithmetically by the smallest natural numbers. Thus the eye, being guided in its estimate by direction rather than by distance, just as the ear is guided by number of vibrations rather than by magnitude, both it and the ear convey simplicity and harmony to the mind without effort, and the mind with equal facility receives and appreciates them.

On the Rectilinear Forms and Proportions of Architecture.

As we are accustomed in all cases to refer direction to the horizontal and vertical lines, and as the meeting of these lines makes the right angle, it naturally constitutes the fundamental angle, by the harmonic division of which a system of proportion may be established, and the theory of symmetrical beauty, like that of music, rendered susceptible of exact reasoning.

Let therefore the right angle be the fundamental angle, and let it be divided upon the quadrant of a circle into the harmonic parts already explained, thus:—

	Right Angle.	Super-tonic Angles.	Mediant Angles.	Sub-dominant Angles.	Dominant Angles.	Sub-mediant Angles.	Sub-tonic Angles.	Semi-sub-tonic Angles.	Tonic Angles.
I.	(1)	$(\frac{8}{9})$	$(\frac{4}{5})$	$(\frac{3}{4})$	$(\frac{2}{3})$	$(\frac{3}{5})$	$(\frac{4}{7})$	$(\frac{8}{15})$	$(\frac{1}{2})$
II.	$(\frac{1}{2})$	$(\frac{4}{9})$	$(\frac{2}{5})$	$(\frac{3}{8})$	$(\frac{1}{3})$	$(\frac{3}{10})$	$(\frac{2}{7})$	$(\frac{4}{15})$	$(\frac{1}{4})$
III.	$(\frac{1}{4})$	$(\frac{2}{9})$	$(\frac{1}{5})$	$(\frac{3}{16})$	$(\frac{1}{6})$	$(\frac{3}{20})$	$(\frac{1}{7})$	$(\frac{2}{15})$	$(\frac{1}{8})$
IV.	$(\frac{1}{8})$	$(\frac{1}{9})$	$(\frac{1}{10})$	$(\frac{3}{32})$	$(\frac{1}{12})$	$(\frac{3}{40})$	$(\frac{1}{14})$	$(\frac{1}{15})$	$(\frac{1}{16})$

In order that the analogy may be kept in view, I have given to the parts of each of these four scales the appropriate nomenclature of the notes which form the diatonic scale in music.

When a right angled triangle is constructed so that its two smallest angles are equal, I term it simply the triangle of $(\frac{1}{2})$, because the smaller angles are each one-half of the right angle. But when the two angles are unequal, the triangle may be named after the smallest. For instance, when the smaller angle, which we shall here suppose to be one-third of the right angle, is made with the vertical line, the triangle may be called the vertical scalene triangle of ($\frac{1}{3}$); and when made with the horizontal line, the horizontal scalene triangle of ($\frac{1}{3}$). As every rectangle is made up of two of these right angled triangles, the same terminology may also be applied to these figures. Thus, the equilateral rectangle or perfect square is simply the rectangle of ($\frac{1}{2}$), being composed of two similar right angled triangles of ($\frac{1}{2}$); and when two vertical scalene triangles of ($\frac{1}{3}$), and of similar dimensions, are united by their hypothenuses, they form the vertical rectangle of ($\frac{1}{3}$), and in like manner the horizontal triangles of ($\frac{1}{3}$) similarly united would form the horizontal rectangle of ($\frac{1}{3}$). As the isosceles triangle is in like manner composed of two right angled scalene triangles joined by one of their sides, the same terminology may be applied to every variety of that figure. All the angles of the first of the above scales, except that of ($\frac{1}{2}$), give rectangles whose longest sides are in the horizontal line, while the other three give rectangles whose longest sides are in the vertical line. I have illustrated in Plate I. the Plate I. manner in which this harmonic law acts upon these elementary rectilinear figures by constructing a series agreeably to the angles of scales II., III., IV. Throughout this series *a b c* is the primary scalene triangle, of which the rectangle *a b c e* is composed; *d c e* the vertical isosceles triangle; and when the plate is turned, *d e a* the horizontal isosceles triangle, both of which are composed of the same primary scalene triangle.

Thus the most simple elements of symmetry in rectilinear forms are the three following figures :—

The equilateral rectangle or perfect square,
The oblong rectangle, and
The isosceles triangle.

It has been shewn that in harmonic combinations of musical sounds, the æsthetic feeling produced by their agreement depends upon the relation they bear to each other with reference to the number of pulsations produced in a given time by the fundamental note of the scale to which they belong; and that the more simply they relate to each other in this way the more perfect the harmony, as in the common chord of the first scale, the relations of whose parts are in the simple ratios of 2 : 1, 3 : 2, and 5 : 4. It is equally consistent with this law, that when applied to form in the composition of an assortment of figures of any kind, their respective proportions should bear a very simple ratio to each other in order that a definite and pleasing harmony may be produced amongst the various parts. Now, this is as effectually done by forming them upon the harmonic divisions of the right angle as musical harmony is produced by sounds resulting from harmonic divisions of a vibratory body.

Having in previous works* given the requisite illustrations of this fact in full detail, I shall here confine myself to the most simple kind, taking for my first example one of the finest specimens of classical architecture in the world—the front portico of the Parthenon of Athens.

The angles which govern the proportions of this beautiful elevation are the following harmonic parts of the right angle—

Tonic Angles	Dominant Angles.	Mediant Angles.	Subtonic Angle.	Supertonic Angles.
$(\frac{1}{2})$	$(\frac{1}{3})$	$(\frac{1}{5})$	$(\frac{1}{7})$	$(\frac{1}{9})$
$(\frac{1}{4})$	$(\frac{1}{6})$	$(\frac{1}{10})$		$(\frac{1}{18})$
$(\frac{1}{8})$				
$(\frac{1}{16})$				

Plate II. In Plate II. I give a diagram of its rectilinear orthography, which is simply constructed by lines drawn, either horizontally,

* "The Geometric Beauty of the Human Figure Defined," &c.

vertically, or obliquely, which latter make with either of the former lines one or other of the harmonic angles in the above series. For example, the horizontal line AB represents the length of the base or surface of the upper step of the substructure of the building. The line AE, which makes an angle of $(\frac{1}{5})$ with the horizontal, determines the height of the colonnade. The line AD, which makes an angle of $(\frac{1}{4})$ with the horizontal, determines the height of the portico, exclusive of the pediment. The line AC, which makes an angle of $(\frac{1}{3})$ with the horizontal, determines the height of the portico, including the pediment. The line GD, which makes an angle of $(\frac{1}{7})$ with the horizontal, determines the form of the pediment. The lines EZ and LY, which respectively make angles of $(\frac{1}{16})$ and $(\frac{1}{18})$ with the horizontal, determine the breadth of the architrave, frieze, and cornice. The line *v n u*, which makes an angle of $(\frac{1}{3})$ with the vertical, determines the breadth of the triglyphs. The line *t d*, which makes an angle of $(\frac{1}{2})$, determines the breadth of the metops. The lines *c b r f*, and *a i*, which make each an angle of $(\frac{1}{6})$ with the vertical, determine the width of the five centre intercolumniations. The line *z k*, which makes an angle of $(\frac{1}{8})$ with the vertical, determines the width of the two remaining intercolumniations. The lines *c s*, *q x*, and *y h*, each of which makes an angle of $(\frac{1}{10})$ with the vertical, determine the diameters of the three columns on each side of the centre. The line *w l*, which makes an angle of $(\frac{1}{9})$ with the vertical, determines the diameter of the two remaining or corner columns.

In all this, the length and breadth of the parts are determined by horizontal and vertical lines, which are necessarily at right angles with each other, and the position of which are determined by one or other of the lines making the harmonic angles above enumerated.

Now, the lengths and breadths thus so simply determined by these few angles, have been proved to be correct by their agreement with the most careful measurements which could possibly be made of this exquisite specimen of formative art. These measurements were obtained by the "Society of Dilettanti," London, who, expressly for that purpose, sent Mr F. C. Penrose, a highly educated architect, to Athens, where he remained for about five months, engaged in the execution of this interesting commission, the results of which are now published in a magnificent volume by the Society.* The agreement was so striking, that Mr Penrose has been publicly thanked by an eminent man of science for bearing testimony to the truth of my theory, who in doing so observes, "The dimensions which he (Mr Penrose) gives are to me the surest verification of the theory I could have desired. The minute discrepancies form that very element of practical incertitude, both as to execution and direct measurement, which always prevails in materialising a mathematical calculation made under such conditions." †

Although the measurements taken by Mr Penrose are undeniably correct, as all who examine the great work just referred to must acknowledge, and although they have afforded me the best possible means of testing the accuracy of my theory as applied to the Parthenon, yet the ideas of Mr Penrose as to the principles they evolve are founded upon the fallacious doctrine which has so long prevailed, and still prevails, in the æsthetics of architecture, viz., that harmony may be imparted by ratios between the lengths and breadths of parts.

I have taken for my second example an elevation which, although of smaller dimensions, is no less celebrated for the beauty of its proportions than the Parthenon itself, viz., the

* Longman and Co., London.

† See Appendix.

front portico of the temple of Theseus, which has also been measured by Mr Penrose.

The angles which govern the proportions of this elevation are the following harmonic parts of the right angle:—

Tonic Angles.	Dominant Angles.	Mediant Angles.
$(\frac{1}{2})$	$(\frac{1}{3})$	$(\frac{2}{5})$
$(\frac{1}{4})$	$(\frac{1}{6})$	$(\frac{1}{5})$
	$(\frac{1}{12})$	

A diagram of the rectilinear orthography of this portico is given in Plate III. Its construction is similar to that of the Parthenon in respect to the harmonic parts of the right angle, and I have therefore only to observe, that the line A E makes an angle of $(\frac{1}{4})$; the line A D an angle of $(\frac{1}{3})$; the line A C an angle of $(\frac{2}{5})$; the line G D an angle of $(\frac{1}{6})$; and the lines E Z and L Y angles of $(\frac{1}{12})$ with the horizontal. Plate III.

As to the colonnade or vertical part, the line *a b*, which determines the three middle intercolumniations, makes an angle of $(\frac{1}{5})$; the line *c d*, which determines the two outer intercolumniations, makes an angle of $(\frac{1}{6})$; and the line *e f*, which determines the lesser diameter of the columns, makes an angle of $(\frac{1}{12})$ with the vertical. I need give no further details here, as my intention is to shew the simplicity of the method by which this theory may be reduced to practice, and because I have given in my other works ample details, in full illustration of the orthography of these two structures, especially the first.*

The foregoing examples being both horizontal rectangular compositions, the proportions of their principal parts have necessarily been determined by lines drawn from the extremities of the base, making angles with the horizontal line, and forming

* "The Orthographic Beauty of the Parthenon," &c., and "The Harmonic Law of Nature applied to Architectural Design."

thereby the diagonals of the various rectangles into which, in their leading features, they are necessarily resolved. But the example I am now about to give is of another character, being a vertical pyramidal composition, and consequently the proportions of its principal parts are determined by the angles which the oblique lines make with the vertical line representing the height of the elevation, and forming a series of isosceles triangles; for the isosceles triangle is the type of all pyramidal composition.

This third example is the east end of Lincoln Cathedral, a Gothic structure, which is acknowledged to be one of the finest specimens of that style of architecture existing in this country.

The angles which govern the proportions of this elevation are the following harmonic parts of the right angle:—

Tonic.	Dominant.	Mediant.	Subtonic.	Supertonic.
$(\frac{1}{2})$	$(\frac{1}{3})$	$(\frac{1}{5})$	$(\frac{1}{7})$	$(\frac{2}{9})$
$(\frac{1}{4})$	$(\frac{1}{6})$	$(\frac{1}{10})$		$(\frac{1}{9})$
	$(\frac{1}{12})$			

Plate IV In Plate IV. I give a diagram of the vertical, horizontal, and oblique lines, which compose the orthography of this beautiful elevation.

The line A B represents the full height of this structure. The line A C, which makes an angle of $(\frac{2}{9})$ with the vertical, determines the width of the design, the tops of the aisle windows, and the bases of the pediments on the inner buttresses; A G, $(\frac{1}{5})$ with the vertical, that of the outer buttress; A F, $(\frac{1}{9})$ with the vertical, that of the space between the outer and inner buttresses and the width of the great centre window; and A E, $(\frac{1}{12})$ with vertical, that of both the inner buttresses and the space between these. A H, which makes $(\frac{1}{4})$ with the vertical, determines the form of the pediment of the centre, and the full height of the base and surbase. A I, which makes ($\frac{1}{3}$) with the vertical, determines the form of the pediment of the smaller gables, the base of the pediment on the outer buttress, the base of the ornamental recess between the outer and inner buttresses, the spring of the arch of the centre window, the tops of the pediments on the inner buttresses, and the spring of the arch of the upper window. A K, which makes ($\frac{1}{2}$), determines the height of the outer buttress; and A Z, which makes ($\frac{1}{6}$) with the horizontal, determines that of the inner buttresses. For the reasons already given, I need not here go into further detail.* It is, however, worthy of remark in this place, that notwithstanding the great difference which exists between the style of composition in this Gothic design, and in that of the east end of the Parthenon, the harmonic elements upon which the orthographic beauty of the one depends, are almost identical with those of the other.

On the Curvilinear Forms and Proportions of Architecture.

Each regular rectilinear figure has a curvilinear figure that exclusively belongs to it, and to which may be applied a corresponding terminology. For instance, the circle belongs to the equilateral rectangle; that is, the rectangle of ($\frac{1}{2}$), an ellipse to every other rectangle, and a composite ellipse to every isosceles triangle. Thus the most simple elements of beauty in the curvilinear forms of architectural design are the following three figures:—

The circle,
The ellipse, and
The composite ellipse.

I find it necessary in this place to go into some details

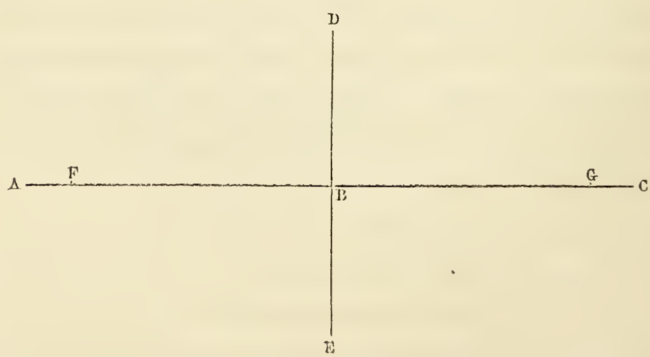
* For further details, see "Harmonic Law of Nature," &c.

regarding the specific character of the two latter figures, because the proper mode of describing these beautiful curves, and their high value in the practice of the architectural draughtsman and ornamental designer, seem as yet unknown. In proof of this assertion, I must again refer to Mr Penrose's great work published by the "Society of Dilettanti." At page 52 of that work it is observed, that "by whatever means an ellipse is to be constructed mechanically, it is a work of time (if not of absolute difficulty) so to arrange the foci, &c., as to produce an ellipse of any exact length and breadth which may be desired." Now, this is far from being the case, for the method of arranging the foci of an ellipse of any given length and breadth is extremely simple, being as follows:—

Let A B C (figure 1) be the length, and D B E the breadth of the desired ellipse.

Take A B upon the compasses, and place the point of one leg upon E and the point of the other upon the line A B, it will meet it at F, which is one focus: keeping the point of the one leg upon E, remove the point of the other to the line B C, and it will meet it at G, which is the other focus.

Fig. 1.

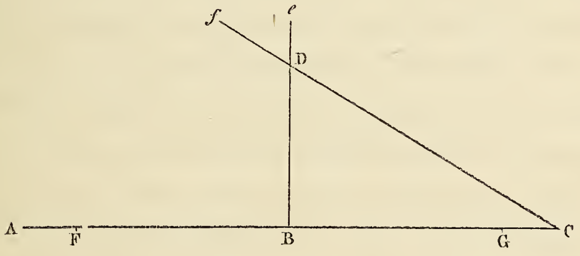


But, when the proportions of an ellipse are to be imparted by means of one of the harmonic angles, suppose the angle of ($\frac{1}{3}$), then the following is the process:—

Let A B C (figure 2) represent the length of the intended ellipse. Through B draw B *e* indefinitely, at right angles with A B C; through C draw the line C *f* indefinitely, making, with B C, an angle of ($\frac{1}{3}$).

Take B C upon the compasses, and place the point of one leg upon D where C *f* intersects B *e*, and the point of the other upon the line A B, it will meet it at F, which is one focus. Keeping the point of one leg still upon D, remove the point of the other to the line B C, and it will meet it at G, which is the other focus.

Fig. 2.



The foci being in either case thus simply ascertained, the method of describing the curve on a small scale is equally simple.

A pin is fixed into each of the two foci, and another into the point D. Around these three pins a waxed thread, flexible but not elastic, is tied, care being taken that the knot be of a kind that will not slip. The pin at D is now removed, and a hard black lead pencil introduced within the thread band. The pencil is then moved around the pins fixed in the foci, keeping the thread band at a full and equal tension;

thus simply the ellipse is described. When, however, the governing angle is acute, say less than ($\frac{1}{6}$), it is requisite to adopt a more accurate method of description,* as the architectural examples which follow will shew. But architectural draughtsmen and ornamental designers would do well to supply themselves, for ordinary practice, with half a dozen series of ellipses, varying in the proportions of their axes from ($\frac{4}{9}$) to ($\frac{1}{6}$) of the scale, and the length of their major axes from 1 to 6 inches. These should be described by the above simple process, upon very strong drawing paper, and carefully cut out, the edge of the paper being kept smooth, and each ellipse having its greater and lesser axes, its foci, and the hypothenuse of its scalene triangle drawn upon
Plate V. it. To exemplify this, I give Plate V., which exhibits the ellipses of ($\frac{1}{3}$), ($\frac{1}{4}$), ($\frac{1}{5}$), and ($\frac{1}{6}$), inscribed in their rectangles, on which *a b* and *c d* are respectively the greater and lesser axes, *o o* the foci, and *d b* the angle of each. Such a series of these beautiful figures would be found particularly useful in drawing the mouldings of Grecian architecture; for, to describe the curvilinear contour of such mouldings from single points, as has been done with those which embellish even our most pretending attempts at the restoration of that classical style of architecture, is to give the resemblance of an external form without the harmony which constitutes its real beauty.

Mr Penrose, owing to the supposed difficulty regarding the description of ellipses just alluded to, endeavours to shew that the curves of all the mouldings throughout the Parthenon

* By a very simple machine, which I have lately invented, an ellipse of any given proportions, even to those of ($\frac{1}{64}$), which is the curve of the entases of the columns of the Parthenon (see Plate VII.), and of any length, from half an inch to fifty feet or upwards, may be easily and correctly described; the length and angle of the required ellipse being all that need be given.

were either parabolic or hyperbolic; but I believe such curves can have no connexion with the elementary forms of architecture, for they are curves which represent motion, and do not, by continued production, form closed figures.

But I have shewn, in a former work,* that the contours of these mouldings are composed of curves of the composite ellipse,—a figure which I so name because it is composed simply of arcs of various ellipses harmonically flowing into each other. The composite ellipse, when drawn systematically upon the isosceles triangle, resembles closely parabolic and hyperbolic curves—only differing from these inasmuch as it possesses the essential quality of circumscribing harmonically one of the elementary rectilinear figures employed in architecture, while those of the parabola and hyperbola, as I have just observed, are merely curves of motion, and, consequently, never can harmonically circumscribe or be resolved into any regular figure.

The composite ellipse may be thus described.

Let A B C (Plate VI.) be a vertical isosceles triangle of Plate VI. ($\frac{1}{6}$), bisect A B in D, and through D draw indefinitely D f perpendicular to A B, and through B draw indefinitely B g, making the angle D B g ($\frac{1}{8}$), D f and B g intersecting each other in M. Take B D and D M as semi-axes of an ellipse, the foci of which will be at p and q, in each of these, and in each of the foci h t and k r in the lines A C and B C, fix a pin, and one also in the point M, tie a thread around these pins, withdraw the pin from M, and trace the composite ellipse in the manner already described with respect to the simple ellipse.

In some of my earlier works I described this figure by taking the angles of the isosceles triangle as foci; but the above method is much more correct. As the elementary angle of

* "The Orthographic Beauty of the Parthenon," &c.

the triangle is ($\frac{1}{6}$), and that of the elliptic curve described around it ($\frac{1}{8}$), I call it the composite ellipse of ($\frac{1}{6}$) and ($\frac{1}{8}$), their harmonic ratio being 4 : 3; and so on of all others, according to the difference that may thus exist between the elementary angles.

The visible curves which soften and beautify the melody of the outline of the front of the Parthenon, as given in Mr Penrose's great work, I have carefully analysed, and have found them in as perfect agreement with this system, as its rectilinear harmony has been shewn to be. This I demonstrated in the work just referred to* by a series of twelve plates, shewing that the entasis of the columns (a subject upon which there has been much speculation) is simply an arc of an ellipse of ($\frac{1}{48}$), whose greater axis makes with the vertical an angle of ($\frac{1}{64}$); or simply, the form of one of these columns is the frustrum of an elliptic-sided or prolate-spheroidal cone, whose section is a composite ellipse of ($\frac{1}{48}$) and ($\frac{1}{64}$), the harmonic ratio of these two angles being 4 : 3, the same as that of the angles of the composite ellipse just exemplified.

Plate VII. In Plate VII. is represented the section of such a cone, of which A B C is the isosceles triangle of ($\frac{1}{48}$), and B D and D M the semi-axes of an ellipse of ($\frac{1}{64}$). M N and O P are the entases of the column, and *d e f* the normal construction of the capital. All these are fully illustrated in the work above referred to,† in which I have also shewn that the curve of the neck of the column is that of an ellipse of ($\frac{1}{6}$); the curve of the capital or echinus, that of an ellipse of ($\frac{1}{14}$); the curve of the moulding under the cymatium of the pediment, that of an ellipse of ($\frac{1}{3}$); and the curve of the bed-moulding of the cornice of the pediment, that of an ellipse of ($\frac{1}{3}$). The curve of the cavetto of the soffit of the corona is composed of ellipses of ($\frac{1}{6}$)

* "The Orthographic Beauty of the Parthenon," &c. † Ibid.

and ($\frac{1}{14}$); the curve of the cymatium which surmounts the corona, is that of an ellipse of ($\frac{1}{3}$); the curve of the moulding of the capital of the antæ of the posticum, that of an ellipse of ($\frac{1}{3}$); the curves of the lower moulding of the same capital are composed of those of an ellipse of ($\frac{1}{3}$) and of the circle ($\frac{1}{2}$); the curve of the moulding which is placed between the two latter is that of an ellipse of ($\frac{1}{3}$); the curve of the upper moulding of the band under the beams of the ceiling of the peristyle, that of an ellipse of ($\frac{1}{3}$); the curve of the lower moulding of the same band, that of an ellipse of ($\frac{1}{4}$); and the curves of the moulding at the bottom of the small step or podium between the columns, are those of the circle ($\frac{1}{2}$) and of an ellipse of ($\frac{1}{3}$). I have also shewn the curve of the fluting of the columns to be that of ($\frac{1}{14}$). The greater axis of each of these ellipses, when not in the vertical or horizontal lines, makes an harmonic angle with one or other of them. In Plate VIII., sections of the two last-named mouldings are Plate VIII.
represented full size, which will give the reader an idea of the simple manner in which the ellipses are employed in the production of those harmonic curves.

Thus we find that the system here adopted for applying this law of nature to the production of beauty in the abstract forms employed in architectural composition, so far from involving us in anything complicated, is characterised by extreme simplicity.

In concluding this part of my treatise, I may here repeat what I have advanced in a late work,* viz., my conviction of the probability that a system of applying this law of nature in architectural construction was the only great practical secret of the Freemasons, all their other secrets being connected, not with their art, but with the social constitution of their society.

* "The Harmonic Law of Nature applied to Architectural Design."

This valuable secret, however, seems to have been lost, as its practical application fell into disuse; but, as that ancient society consisted of speculative as well as practical masons, the secrets connected with their social union have still been preserved, along with the excellent laws by which the brotherhood is governed. It can scarcely be doubted that there was some such practically useful secret amongst the Freemasons or early Gothic architects; for we find in all the venerable remains of their art which exist in this country, symmetrical elegance of form pervading the general design, harmonious proportion amongst all the parts, beautiful geometrical arrangements throughout all the tracery, as well as in the elegantly symmetrised foliated decorations which belong to that style of architecture. But it is at the same time worthy of remark, that whenever they diverged from architecture to sculpture and painting, and attempted to represent the human figure, or even any of the lower animals, their productions are such as to convince us that in this country these arts were in a very degraded state of barbarism—the figures are often much disproportioned in their parts and distorted in their attitudes, while their representations of animals and chimeras are whimsically absurd. It would, therefore, appear that achitecture, as a fine art, must have been preserved by some peculiar influence from partaking of the barbarism so apparent in the sister arts of that period. Although its practical secrets have been long lost, the Freemasons of the present day trace the original possession of them to Moses, who, they say, "modelled masonry into a perfect system, and circumscribed its mysteries by *land-marks* significant and unalterable." Now, as Moses received his education in Egypt, where Pythagoras is said to have acquired his first knowledge of the harmonic law of numbers, it is highly probable that this perfect system of the great Jewish legislator was based upon the same law of nature which constituted the foundation of the Pythagorean philosophy, and ultimately led to that excellence in art which is still the admiration of the world.

Pythagoras, it would appear, formed a system much more perfect and comprehensive than that practised by the Freemasons in the middle ages of Christianity; for it was as applicable to sculpture, painting, and music, as it was to architecture. This perfection in architecture is strikingly exemplified in the Parthenon, as compared with the Gothic structures of the middle ages; for it will be found that the whole six elementary figures I have enumerated as belonging to architecture, are required in completing the orthographic beauty of that noble structure. And amongst these, none conduce more to that beauty than the simple and composite ellipses. Now, in the architecture of the best periods of Gothic, or, indeed, in that of any after period (Roman architecture included), these beautiful curves seem to have been ignored, and that of the circle alone employed.

Be those matters as they may, however, the great law of numerical harmonic ratio remains unalterable, and a proper application of it in the science of art will never fail to be as productive of effect, as its operation in nature is universal, certain, and continual.

THE SCIENCE OF BEAUTY, AS DEVELOPED IN THE HUMAN HEAD AND COUNTENANCE.

The most remarkable characteristics of the human head and countenance are the globular form of the cranium, united as it is with the prolate spheroidal form produced by the parts which constitute the face, and the approximation of the profile to the vertical; for in none of the lower animals does the skull present so near a resemblance to a combination of these geometric forms, nor the plane of the face to this direction. We also find that although these peculiar characteristics are variously modified among the numerous races of mankind, yet one law appears to govern the beauty of the whole. The highest and most cultivated of these races, however, present only an approximation to the perfect development of those distinguishing marks of humanity; and therefore the beauty of form and proportion which in nature characterises the human head and countenance, exhibits only a partial development of the harmonic law of visible beauty. On the other hand, we find that, in their sculpture, the ancient Greeks surpassed ordinary nature, and produced in their beau ideal a species of beauty free from the imperfections and peculiarities that constitute the individuality by which the countenances of men are distinguished from each other. It may be requisite here to remark, that this species of beauty is independent of the more intellectual quality of expression. For as Sir Charles Bell has said, "Beauty of countenance may be defined in words, as well as demonstrated in art. A face may be beautiful in sleep, and a statue without expression may be highly beautiful. But it will be said there is expression in the sleeping figure or in the statue. Is it not rather that we see in these the capacity for expression?—that our minds are active in imagining what may be the motions of these features when awake or animated? Thus, we speak of an expressive face before we have seen a movement grave or cheerful, or any indication in the features of what prevails in the heart."

This capacity for expression certainly enhances our admiration of the human countenance; but it is more a concomitant of the primary cause of its beauty than the cause itself. This cause rests on that simple and secure basis—the harmonic law of nature; for the nearer the countenance approximates to an harmonious combination of the most perfect figures in geometry, or rather the more its general form and the relation of its individual parts are arranged in obedience to that law, the higher its degree of beauty, and the greater its capacity for the expression of the passions.

Various attempts have been made to define geometrically the difference between the ordinary and the ideal beauty of the human head and countenance, the most prominent of which is that of Camper. He traced, upon a profile of the skull, a line in a horizontal direction, passing through the foramen of the ear and the exterior margin of the sockets of the front teeth of the upper jaw, upon which he raised an oblique line, tangential to the margin of these sockets, and to the most prominent part of the forehead. Agreeably to the obliquity of this line, he determined the relative proportion of the areas occupied by the brain and by the face, and hence inferred the degree of intellect. When he applied this measurement to the heads of the antique statues, he found the angle much greater than in ordinary nature ; but that this simple fact afforded no rule for the reproduction of the ideal beauty of ancient Greek art, is very evident from the heads and countenances by which his treatise is illustrated. Sir Charles Bell justly remarks, that although, by Camper's method, the forehead may be thrown forward, yet, while the features of common nature are preserved, we refuse to acknowledge a similarity to the beautiful forms of the antique marbles. "It is true," he says, "that, by advancing the forehead, it is raised, the face is shortened, and the eye brought to the centre of the head. But with all this, there is much wanting—that which measurement, or a mere line, will not shew us."—"The truth is, that we are more moved by the features than by the form of the whole head. Unless there be a conformity in every feature to the general shape of the head, throwing the forehead forward on the face produces deformity ; and the question returns with full force—How is it that we are led to concede that the antique head of the Apollo, or of the Jupiter, is beautiful when the facial line makes a hundred degrees with the horizontal line ? In other words—How do we admit that to be beautiful which is not natural ? Simply for the same reason that, if we discover a broken portion of an antique, a nose, or a chin of marble, we can say, without deliberation—This must have belonged to a work of antiquity ; which proves that the character is distinguishable in every part —in each feature, as well as in the whole head."

Dr Oken says upon this subject : *—"The face is beautiful whose nose is parallel to the spine. No human face has grown

* "Physio-philosophy." By Dr Oken. Translated by Talk ; and published by the Ray Society. London, 1848.

into this estate ; but every nose makes an acute angle with the spine. The facial angle is, as is well known, 80°. What, as yet, no man has remarked, and what is not to be remarked, either, without our view of the cranial signification, the old masters have felt through inspiration. They have not only made the facial angle a right angle, but have even stepped beyond this—the Romans going up to 96°, the Greeks even to 100°. Whence comes it that this unnatural face of the Grecian works of art is still more beautiful than that of the Roman, when the latter comes nearer to nature? The reason thereof resides in the fact of the Grecian artistic face representing nature's design more than that of the Roman; for, in the former, the nose is placed quite perpendicular, or parallel to the spinal cord, and thus returns whither it has been derived."

Other various and conflicting opinions upon this subject have been given to the world; but we find that the principle from which arose the ideal beauty of the head and countenance, as represented in works of ancient Greek art, is still a matter of dispute. When, however, we examine carefully a fine specimen, we find its beauty and grandeur to depend more upon the degree of harmony amongst its parts, as to their relative proportions and mode of arrangement, than upon their excellence taken individually. It is, therefore, clear that those (and they are many) who attribute the beauty of ancient Greek sculpture merely to a selection of parts from various models, must be in error. No assemblage of parts from ordinary nature could have produced its principal characteristic, the excess in the angle of the facial line, much less could it have led to that exquisite harmony of parts by which it is so eminently distinguished; neither can we reasonably agree with Dr Oken and others, who assert that it was produced by an exclusive degree of the inspiration of genius amongst the Greek people during a certain period.

That the inspiration of genius, combined with a careful study of nature, were essential elements in the production of the great works which have been handed down to us, no one will deny; but these elements have existed in all ages, whilst the ideal head belongs exclusively to the Greeks during the period in which the schools of Pythagoras and Plato were open. Is it not, therefore, reasonable to suppose, that, besides genius and the study of nature, another element was employed in the production of this excellence, and that this element arose from the precise mathematical doctrines taught in the schools of these philosophers?

An application of the great harmonic law seems to prove that there is no object in nature in which the science of beauty is more clearly developed than in the human head and countenance, nor to the representations of which the same science is more easily applied; and it is to the mode in which this is done that the varieties of sex and character may be imparted to works of art. Having gone into full detail, and given ample illustrations in a former work,* it is unnecessary for me to enter upon that part of the subject in this *résumé;* but only to shew the typical structure of beauty by which this noble work of creation is distinguished.

The angles which govern the form and proportions of the human head and countenance are, with the right angle, a series of seven, which, from the simplicity of their ratios to each other, are calculated to produce the most perfect concord. It consists of the right angle and its following parts—

* "The Science of those Proportions by which the Human Head and Countenance, as represented in Works of ancient Greek Art, are distinguished from those of ordinary Nature."

Tonic.	Dominant.	Mediant.	Subtonic.
$(\frac{1}{2})$	$(\frac{1}{3})$	$(\frac{1}{5})$	$(\frac{1}{7})$
$(\frac{1}{4})$	$(\frac{1}{6})$		

These angles, and the figures which belong to them, are thus arranged:—

The vertical line A B (Plate IX. fig. 2) represents the full length of the head and face. Taking this line as the greater axis of an ellipse of $(\frac{1}{3})$, such an ellipse is described around it. Through A the lines A G, A K, A L, A M, and A N, are drawn on each side of the line A B, making, with the vertical, respectively the angles of $(\frac{1}{3})$, $(\frac{1}{4})$, $(\frac{1}{5})$, $(\frac{1}{6})$, and $(\frac{1}{7})$. Through the points G, K, L, M, and N, where these straight lines meet the curved line of the ellipse, horizontal lines are drawn by which the following isosceles triangles are formed, A G G, A K K, A L L, A M M, and A N N. From the centre X of the equilateral triangle A G G the curvilinear figure of $(\frac{1}{2})$, viz., the circle, is described circumscribing that triangle. Plate IX.

The curvilinear plane figures of $(\frac{1}{2})$ and $(\frac{1}{3})$, respectively, represent the solid bodies of which they are sections, viz., a sphere and a prolate spheroid. These bodies, from the manner in which they are here placed, are partially amalgamated, as shewn in figures 1 and 3 of the same plate, thus representing the form of the human head and countenance, both in their external appearance and osseous structure, more correctly than they could be represented by any other geometrical figures. Thus, the angles of $(\frac{1}{2})$ and $(\frac{1}{3})$ determine the typical form.

From each of the points *u* and *n*, where A M cuts G G on both sides of A B, a circle is described through the points *p* and *q*, where A K cuts G G on both sides of A B, and with the same radius a circle is described from the point *a*, where K K cuts A B.

The circles *u* and *n* determine the position and size of the eyeballs, and the circle *a* the width of the nose, as also the horizontal width of the mouth.

The lines G G and K K also determine the length of the joinings of the ear to the head. The lines L L and M M determine the vertical width of the mouth and lips when at perfect repose, and the line N N the superior edge of the chin. Thus simply are the features arranged and proportioned on the facial surface.

It must, however, be borne in mind, that in treating simply of the æsthetic beauty of the human head and countenance, we have only to do with the external appearance. In this research, therefore, the system of Dr Camper, Dr Owen, and others, whose investigations were more of a physiological than an æsthetic character, can be of little service; because, according to that system, the facial angle is determined by drawing a line tangential to the exterior margin of the sockets of the front teeth of the upper jaw, and the most prominent part of the forehead. Now, as these sockets are, when the skull is naturally clothed, and the features in repose, entirely concealed by the upper lip, we must take the prominent part of it, instead of the sockets under it, in order to determine properly this distinguishing mark of humanity. And I believe it will be found, that when the head is properly poised, the nearer the angle which this line makes with the horizontal approaches 90°, the more symmetrically beautiful will be the general arrangement of the parts (see line *y z*, figure 3, Plate IX.).

THE SCIENCE OF BEAUTY, AS DEVELOPED IN THE FORM OF THE HUMAN FIGURE.

The manner in which this science is developed in the symmetrical proportions of the entire human figure, is as remarkable for its simplicity as it has been shewn to be in those of the head and countenance. Having gone into very full details, and given ample illustration in two former works * upon this subject, I may here confine myself to the illustration of one description of figure, and to a reiteration of some facts stated in these works. These facts are, 1*st*, That on a given line the human figure is developed, as to its principal points, entirely by lines drawn either from the extremities of this line, or from some obvious or determined localities. 2*d*, That the angles which these lines make with the given line, are all simple sub-multiples of some given fundamental angle, or bear to it a proportion expressible under the most simple relations, such as those which constitute the scale of music. 3*d*, That the contour is resolved into a series of ellipses of the same simple angles. And, 4*th*, That these ellipses, like the lines, are inclined to the first given line by angles which are simple sub-multiples of the given fundamental angle.

* "The Geometric Beauty of the Human Figure Defined," &c., and "The Natural Principles of Beauty Developed in the Human Figure."

From which four facts, and agreeably to the hypothesis I have adopted, it results as a natural consequence that the only effort which the mind exercises through the eye, in order to put itself in possession of the data for forming its judgment, is this, that it compares the angles about a point, and thereby appreciates the simplicity of their relations. In selecting the prominent features of a figure, the eye is not seeking to compare their relative distances—it is occupied solely with their relative positions. In tracing the contour, in like manner, it is not left in vague uncertainty as to what is the curve which is presented to it; unconsciously it feels the complete ellipse developed before it; and if that ellipse and its position are both formed by angles of the same simple relative value as those which aided its determination of the positions of the prominent features, it is satisfied, and finds the symmetry perfect.

Müller, and other investigators into the archæology of art, refer to the great difficulty which exists in discovering the principles which the ancients followed in regard to the proportions of the human figure, from the different sexes and characters to which they require to be applied. But in the system thus founded upon the harmonic law of nature, no such difficulty is felt, for it is as applicable to the massive proportions which characterise the ancient representations of the Hercules, as to the delicate and perfectly symmetrical beauty of the Venus. This change is effected simply by an increase in the fundamental angle. For instance, in the construction of a figure of the exact proportions of the Venus, the right angle is adopted. But in the construction of a figure of the massive proportions of the Hercules, it is requisite to adopt an angle which bears to the right angle the ratio of 6 : 5. The adoption of this angle I have shewn in another work * to produce in the Hercules those proportions which are so characteristic of physical power. The ellipses which govern the outline, being also formed upon the same larger class of angles, give the contour of the muscles a more massive character. In comparing the male and female forms thus geometrically constructed, it will be found that that of the female is more harmoniously symmetrical, because the right angle is the fundamental angle for the trunk and the limbs as well as for the head and countenance; while in that of the male, the right angle is the fundamental angle for the head only. It may also be observed, that, from the greater proportional width of the pelvis of the female, the centres of that motion which the heads of the thigh bones perform in the cotyloid cavities, and the centres of that still more extensive range of motion which the arm is capable of performing at the shoulder joints, are nearly in the same line which determines the central motion of the vertribal column, while those of the male are not; consequently all the motions of the female are more graceful than those of the male.

This difference between the fundamental angles, which impart to the human figure, on the one hand, the beauty of feminine proportion and contour, and on the other, the grandeur of masculine strength, being in the ratio of 5 : 6, allows ample latitude for those intermediate classes of proportions which the ancients imparted to their various other deities in which these two qualities were blended. I therefore confine myself to an illustration of the external contour of the form, and the relative proportions of all the parts of a female figure, such as those of the statues of the Venus of Melos and Venus of Medici.

* "The Geometric Beauty of the Human Figure Defined," &c.

The angles which govern the form and proportions of such a figure are, with the right angle, a series of twelve, as follows:—

Tonic.	Dominant.	Mediant.	Subtonic.	Supertonic.
$(\frac{1}{2})$	$(\frac{1}{3})$	$(\frac{1}{5})$	$(\frac{1}{7})$	$(\frac{1}{9})$
$(\frac{1}{4})$	$(\frac{1}{6})$	$(\frac{1}{10})$	$(\frac{1}{14})$	
$(\frac{1}{8})$	$(\frac{1}{12})$			

These angles are employed in the construction of a diagram, which determines the proportions of the parts throughout the whole figure. Thus:—

Plate X. Let the line A B (fig. 1, plate X.) represent the height of the figure to be constructed. At the point A, make the angles of C A D $(\frac{1}{3})$, F A G $(\frac{1}{4})$, H A I $(\frac{1}{5})$, K A L $(\frac{1}{6})$, and M A N $(\frac{1}{7})$. At the point B, make the angles K B L $(\frac{1}{8})$, U B A $(\frac{1}{12})$, and O B A $(\frac{1}{14})$.

Through the point K, in which the lines A K and B K intersect one another, draw P K O parallel to A B, and through C F H and M, where this line meets A C, A F, A H, and A M, draw C D, F G, H I, and M N, perpendicular to A B; draw also K L perpendicular to A B; join B F and B H, and through C draw C E, making with A B the angle $(\frac{1}{2})$, which completes the arrangement of the eleven angles upon A B; for F B G is very nearly $(\frac{1}{10})$, and H B I very nearly $(\frac{1}{9})$.

At the point *f*, where A C intersects O B, draw *f a* perpendicular to A B; and through the point *i*, where B O intersects M N, draw S *i* T parallel to A C.

Through *m*, where S *i* T intersects F B, draw *m n;* through *β*, where S *i* T intersects K B, draw *β w;* through T draw T *g*, making an angle of $(\frac{1}{3})$ with O P. Join N P, M B, and *g* P, and where N P intersects K B, draw Q R perpendicular to A B.

On A E as a diameter, describe a circle cutting A C in *r*, and draw *r o* perpendicular to A B.

With A *o* and *o r* as semi-axes, describe the ellipse A *r e*, cutting A H in *t;* and draw *t u* perpendicular to A B. With A *u* and *t u*, as semi-axes describe the ellipse A *t d*. On *a* L, as major axis, describe the ellipse of ($\frac{1}{3}$).

For the side aspect or profile of the figure the diagram is thus constructed—

On one side of a line A B (fig. 2, Plate X.) construct the rectilinear portion of a diagram the same as fig. 1. Through *i* draw W Y parallel to A B, and draw A *z* perpendicular to A B. Make W *a* equal to A *a* (fig. 1), and on *a l*, as major axis, describe the ellipse of ($\frac{1}{4}$). Through *a* draw *a p* parallel to A F, and through *p* draw *p t* perpendicular to W Y. Through *a* draw *f a u* perpendicular to W Y.

Upon a diameter equal to A E describe a circle whose circumference shall touch A B and A *z*. With semi-axes equal to A *o* and *o r* (fig. 1), describe an ellipse with its major axis parallel to A B, and its circumference touching O P and *z* A.

Thus simply are the diagrams of the general proportions of the human figure, as viewed in front and in profile, constructed; and Plate XI. gives the contour in both points of view, as Plate XI. composed entirely of the curvilinear figures of ($\frac{1}{2}$), ($\frac{1}{3}$), ($\frac{1}{4}$), ($\frac{1}{5}$,) and ($\frac{1}{6}$).

Further detail here would be out of place, and I shall therefore refer those who require it to the Appendix, or the more elaborate works to which I have already referred.

The beauty derived from proportion, imparted by the system here pointed out, and from a contour of curves derived from the same harmonic angles, is not confined to the human figure, but is found in various minor degrees of perfection in all the organic forms of nature, whether animate or inanimate, of which I have in other works given many examples.*

* "Essay on Ornamental Design," &c., and "The Geometric Beauty of the Human Figure," &c.

THE SIENCE OF BEAUTY, AS DEVELOPED IN COLOURS.

There is not amongst the various phenomena of nature one that more readily excites our admiration, or makes on the mind a more vivid impression of the order, variety, and harmonious beauty of the creation, than that of colour. On the general landscape this phenomenon is displayed in the production of that species of harmony in which colours are so variously blended, and in which they are by light, shade, and distance modified in such an infinity of gradation and hue. Although genius is continually struggling, with but partial success, to imitate those effects, yet, through the Divine beneficence, all whose organs of sight are in an ordinary degree of perfection can appreciate and enjoy them. In winter this pleasure is often to a certain extent withdrawn, when the colourless snow alone clothes the surface of the earth. But this is only a pause in the general harmony, which, as the spring returns, addresses itself the more pleasingly to our perception in its vernal melody, which, gradually resolving itself into the full rich hues of luxuriant beauty exhibited in the foliage and flowers of summer, subsequently rises into the more vivid and powerful harmonies of autumn's colouring. Thus the eye is prepared again to enjoy that rest which such exciting causes may be said to have rendered necessary.

When we pass from the general colouring of nature to that of particular objects, we are again wrapt in wonder and admiration by the beauty and harmony which so constantly, and in such infinite variety, present themselves to our view, and which are so often found combined in the most minute objects. And the systematic order and uniformity perceptible amidst this endless variety in the colouring of animate and inanimate nature is thus another characteristic of beauty equally prevalent throughout creation.

By this uniformity in colour, various species of animals are often distinguished; and in each individual of most of these species, how much is this beauty enhanced when the uniformity prevails in the resemblance of their lateral halves! The human countenance exemplifies this in a striking manner; the slightest variety of colour between one and another of the double parts is at once destructive of its symmetrical beauty. Many of the lower animals, whether inhabitants of the earth, the air, or the water, owe much of their beauty to this kind of uniformity in the colour of the furs, feathers, scales, or shells, with which they are clothed.

In the vegetable kingdom, we find a great degree of uniformity of colour in the leaves, flowers, and fruit of the same plant, combined with all the harmonious beauty of variety which a little careful examination develops.

In the colours of minerals, too, the same may be observed. In short, in the beauty of colouring, as in every other species of beauty, uniformity and variety are found to combine.

An appreciation of colour depends, in the first place, as much upon the physical powers of the eye in conveying a proper impression to the mind, as that of music on those of the ear. But an ear for music, or an eye for colour, are, in so far as beauty is concerned, erroneous expressions; because they are merely applicable to the impression made upon the senses, and do not refer to the æsthetical principles of harmony, by which beauty can alone be understood.

A good eye, combined with experience, may enable us to form a correct idea as to the purity of an individual colour, or of the relative difference existing between two separate hues; but this sort of discrimination does not constitute that kind of appreciation of the harmony of colour by which we admire and enjoy its development in nature and art. The power of perceiving and appreciating beauty of any kind, is a principle inherent in the human mind, which may be improved by cultivation in the degree of the perfection of the art senses. Great pains have been bestowed on the education of the ear, in assisting it to appreciate the melody and harmony of sound; but still much remains to be done in regard to the cultivation of the eye, in appreciating colour as well as form.

It is true, that there are individuals whose powers of vision are perfect, in so far as regards the appreciation of light, shade, and configuration, but who are totally incapable of perceiving effects produced by the intermediate phenomenon of colour, every object appearing to them either white, black, or neutral gray; others, who are equally blind as to the effect of one of the three primary colours, but see the other two perfectly, either singly or combined; while there are many who, having the full physical power of perceiving all the varieties of the phenomenon, and who are even capable of making nice distinctions amongst a variety of various colours, are yet incapable of appreciating the æsthetic quality of harmony which exists in their proper combination. It is the same with respect to the effects of sounds upon the ear—some have organs so constituted, that notes above or below a certain pitch are to them inaudible; while others, with physical powers otherwise perfect, are incapable of appreciating either melody or harmony in musical composition. But perceptions so imperfectly constituted are, by the goodness of the Creator, of very rare occurrence; therefore all attempts at improvement in the science of æsthetics must be suited to the capacities of the generality of mankind, amongst whom the perception of colour exists in a variety as great as that by which their countenances are distinguished. Artists now and then appear who have this intuitive perception in such perfection, that they are capable of transferring to their works the most beautiful harmonies and most delicate gradations of colours, in a manner that no acquired knowledge could have enabled them to impart. To those who possess such a gift, as well as to those to whom the ordinary powers of perception are denied, it would be equally useless to offer an explanation of the various modes in which the harmony of colour develops itself, or to attempt a definition of the many various colours, hues, tints, and shades, arising out of the simple elements of this phenomenon. But to those whose powers lie between these extremes, being neither above nor below cultivation, such an explanation and definition must form a step towards the improvement of that inherent principle which constitutes the basis of æsthetical science.

Although the variety and harmony of colour which nature is continually presenting to our view, are apparent to all whose visual organs are in a natural state, and thus to the generality of mankind; yet a knowledge of the simplicity by which this variety and beauty are produced, is, after ages of philosophic research and experimental inquiry, only beginning to be properly understood.

Light may be considered as an active, and darkness a passive principle in the economy of Nature, and colour an intermediate phenomenon arising from their joint influence;

and it is in the ratios in which these primary principles act upon each other, by which I here intend to explain the science of beauty as evolved in colour. It has been usual to consider colour as an inherent quality in light, and to suppose that coloured bodies absorb certain classes of its rays, and reflect or transmit the remainder; but it appears to me that colour is more probably the result of certain modes in which the opposite principles of motion and rest, or force and resistance, operate in the production, refraction, and reflection of light, and that each colour is mutually related, although in different degrees, to these active and passive principles.

White and black are the representatives of light and darkness, or activity and rest, and are therefore calculated as pigments to reduce colours and hues to tints and shades.

Having, however, fully illustrated the nature of tints and shades in a former work,* I shall here confine myself to colours in their full intensity—shewing the various modifications which their union with each other produce, along with the harmonic relations which these modifications bear to the primaries, and to each other in respect to warmth and coolness of tone, as well as to light and shade.

The primary colours are red, yellow, and blue. Of these, yellow is most allied to light, and blue to shade, while red is neutral in these respects, being equally allied to both. In respect to tone, that of red is warm, and that of blue cool, while the tone of yellow is neutral. The ratios of their relations to each other in these respects will appear in the harmonic scales to which, for the first time, I am about to subject colours, and to systematise their various simple and compound relations, which are as follow:—

* "A Nomenclature of Colours, applicable to the Arts and Natural Sciences," &c., &c.

From the binary union of the primary colours, the secondary colours arise—

Orange colour, from the union of yellow and red.

Green, from the union of yellow and blue.

Purple, from the union of red and blue.

From the binary union of the secondary colours, the primary hues arise—

Yellow-hue, from the union of orange and green.

Red-hue, from the union of orange and purple.

Blue-hue, from the union of purple and green.

From the binary union of the primary hues, the secondary hues arise—

Orange-hue, from the union of yellow-hue and red-hue.

Green-hue, from the union of yellow-hue aud blue-hue.

Purple-hue, from the union of red-hue and blue-hue.

Each hue owes its characteristic distinction to the proportionate predominance or subordination of one or other of the three primary colours in its composition.

It follows, that in every hue of *red*, yellow and blue are subordinate ; in every hue of *yellow*, red and blue are subordinate ; and in every hue of *blue*, red and yellow are subordinate. In like manner, in every hue of *green*, red is subordinate ; in every hue of *orange*, blue is subordinate ; and in every hue of *purple*, yellow is subordinate.

By the union of two primary colours, in the production of a secondary colour, the nature of both primaries is altered ; and as there are only three primary or simple colours in the scale, the two that are united harmonically in a compound colour, form the natural contrast to the remaining simple colour.

Notwithstanding all the variety that extends beyond the six positive colours, it may be said that there are only three proper contrasts of colour in nature, and that all others are simply modifications of these.

Pure red is the most perfect contrast to pure green; because it is characterised amongst the primary colours by warmth of tone, while amongst the secondary colours green is distinguished by coolness of tone, both being equally related to the primary elements of light and shade.

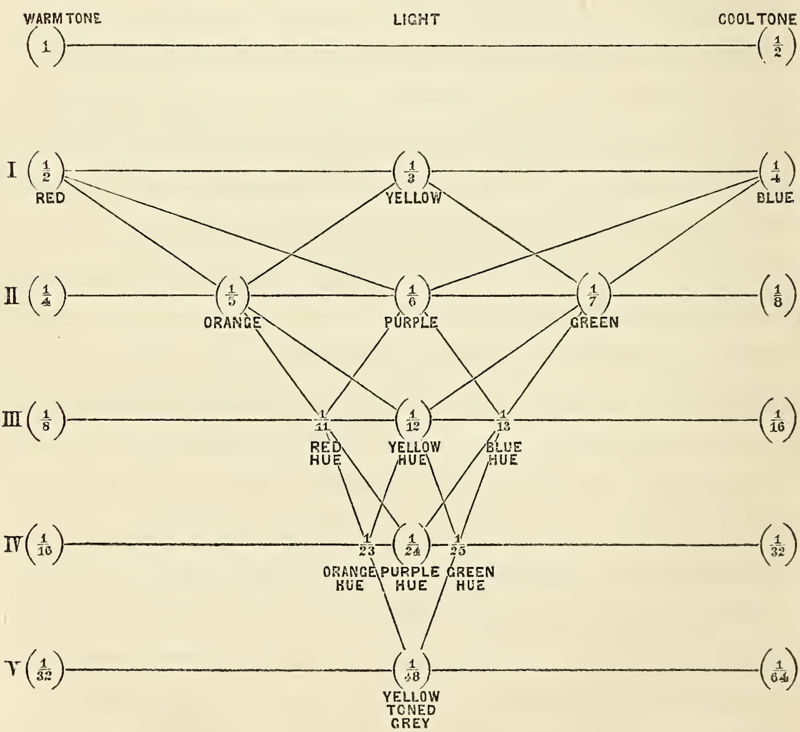
Pure yellow is the most perfect contrast to pure purple; because it is characterised amongst the primary colours as most allied to light, whilst pure purple is characterised amongst the secondaries as most allied to shade, both being equally neutral as to tone.

Pure blue is the most perfect contrast to pure orange; because it is characterised amongst the primary colours as not only the most allied to shade, but as being the coolest in tone, whilst pure orange is characterised amongst the secondaries as being the most allied to light and the warmest in tone. The same principle operates throughout all the modifications of these primary and secondary colours.

Such is the simple nature of contrast upon which the beauty of colouring mainly depends.

It being now established as a scientific fact, that the effect of light upon the eye is the result of an ethereal action, similar to the atmospheric action by which the effect of sound is produced upon the ear; also, that the various colours which light assumes are the effect of certain modifications in this ethereal action;—just as the various sounds, which constitute the scale of musical notes, are known to be the effect of certain modifications in the atmospheric action by which sounds in general are produced:

Therefore, as harmony may thus be impressed upon the mind through either of these two art senses—hearing and seeing—the principles which govern the modifications in the ethereal action of light, so as to produce through the eye the effect of harmony, cannot be supposed to differ from those principles which we know govern the modifications of the atmospheric action of sound, in producing through the ear a like effect. I shall therefore endeavour to illustrate the science of beauty as evolved in colours, by forming scales of their various modifications agreeably to the same Pythagorean system of numerical ratio from which the harmonic elements of beauty in sounds were originally evolved, and by which I have endeavoured in this, as in previous works, to systematise the harmonic beauty of forms.



It will be observed, that with a view to avoid complexity as much as possible, I have, in the arrangement of the above series of scales, not only confined myself to the merely elementary parts of the Pythagorean system, but have left out the harmonic modifications upon ($\frac{1}{11}$) and ($\frac{1}{13}$), in order that the arithmetical progression might not be interrupted.*

The above elementary process will, I trust, be found sufficient to explain the progress, by harmonic union, of a primary colour to a toned gray, and how the simple and compound colours naturally arrange themselves into the elements of five scales, the parts of which continue from primary to secondary colour; from secondary colour to primary hue; from primary hue to secondary hue; from secondary hue to primary-toned gray; and from primary-toned gray to secondary-toned gray in the simple ratio of 2 : 1; thereby producing a series of the most beautiful and perfect contrasts.

The natural arrangement of the primary colours upon the solar spectrum is red, yellow, blue, and I have therefore adopted the same arrangement on the present occasion. Red being, consequently, the first tonic, and blue the second, the divisions express the numerical ratios which the colours bear to one another, in respect to that colourific power for which red is pre-eminent. Thus, yellow is to red, as 2 : 3; blue to yellow, as 3 : 4; purple to orange, as 5 : 6; and green to purple, as 6 : 7.

The following series of completed scales are arranged upon the foregoing principle, with the natural connecting links of red-orange, yellow-orange, yellow-green, and blue-green, introduced in their proper places.

The appropriate terminology of musical notes has been adopted, and the scales are composed as follows:—

* See pp. 24 and 25.

Scale I. consists of primary and secondary colours;
Scale II. of secondary colours and primary hues;
Scale III. of primary and secondary hues;
Scale IV. of secondary hues and primary-toned grays; and
Scale V. of primary and secondary-toned grays;

All the parts in each of these scales, from the first tonic to the second, relate to the same parts of the scale below them in the simple ratio of 2 : 1; and serially to the first tonic in the following ratios:—

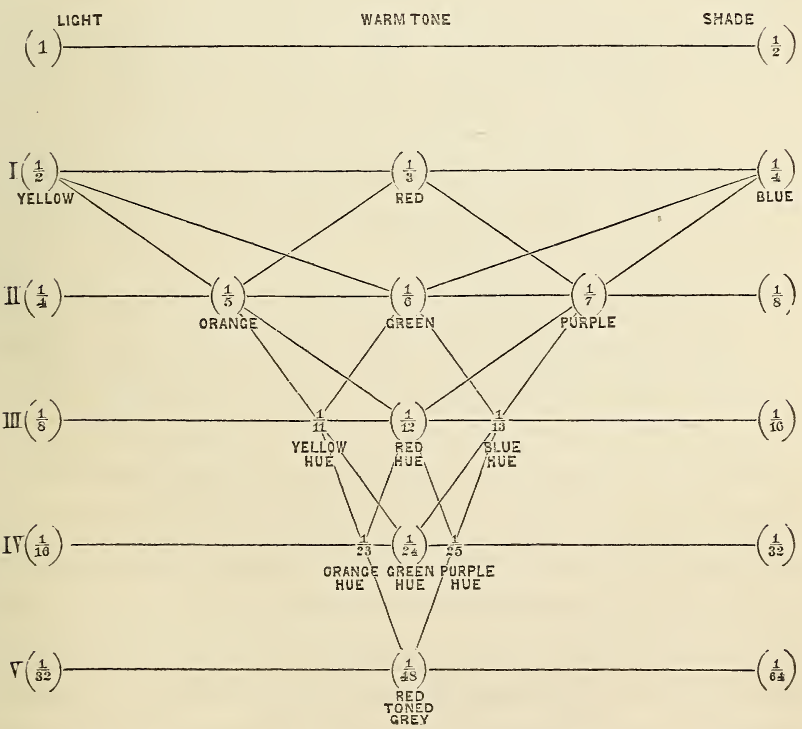
8 : 9, 4 : 5, 3 : 4, 2 : 3, 3 : 5, 4 : 7, 8 : 15, 1 : 2.

First Series of Scales.

	Tonic.	Super-tonic.	Mediant.	Sub-dominant.	Dominant.	Sub-mediant.	Sub-tonic.	Semi-Subtonic.	Tonic.
I.	($\frac{1}{2}$) Red.	($\frac{4}{9}$) Red-orange.	($\frac{2}{5}$) Orange.	($\frac{3}{8}$) Yellow-orange.	($\frac{1}{3}$) Yellow.	($\frac{3}{10}$) Yellow-green.	($\frac{2}{7}$) Green.	($\frac{4}{15}$) Blue-green.	($\frac{1}{4}$) Blue.
II.	($\frac{1}{4}$) Green.	($\frac{2}{9}$) Blue-green hue.	($\frac{1}{5}$) Blue hue.	($\frac{3}{16}$) Blue-purple hue.	($\frac{1}{6}$) Purple.	($\frac{3}{20}$) Red-purple hue.	($\frac{1}{7}$) Red hue.	($\frac{2}{15}$) Red-orange hue.	($\frac{1}{8}$) Orange.
III	($\frac{1}{8}$) Red hue.	($\frac{1}{9}$) Red-orange hue.	($\frac{1}{10}$) Orange hue.	($\frac{3}{32}$) Yellow-orange hue.	($\frac{1}{12}$) Yellow hue.	($\frac{3}{40}$) Yellow-green hue.	($\frac{1}{14}$) Green hue.	($\frac{1}{15}$) Blue-green hue.	($\frac{1}{16}$) Blue hue.
IV.	($\frac{1}{16}$) Green hue.	($\frac{1}{18}$) Blue-green-toned gray.	($\frac{1}{20}$) Blue-toned gray.	($\frac{3}{64}$) Blue-purple-toned gray.	($\frac{1}{24}$) Purple hue.	($\frac{3}{80}$) Red-purple-toned gray.	($\frac{1}{28}$) Red-toned gray.	($\frac{1}{30}$) Red-orange-toned gray.	($\frac{1}{32}$) Orange hue.
V.	($\frac{1}{32}$) Red-toned gray.	($\frac{1}{36}$) Red-orange-toned gray.	($\frac{1}{40}$) Orange-toned gray.	($\frac{3}{128}$) Yellow-orange-toned gray.	($\frac{1}{48}$) Yellow-toned gray.	($\frac{3}{160}$) Yellow-green-toned gray.	($\frac{1}{56}$) Green-toned gray.	($\frac{1}{60}$) Blue-green-toned gray.	($\frac{1}{64}$) Blue-toned gray.

To the scales of chromatic power I add another series of scales, in which yellow, being the first tonic, and blue the second, the numerical divisions express the ratios which the colours in each scale bear to one another in respect to light and shade. Thus red is to yellow, in respect to light, as 2 : 3; blue to red, as 3 : 4; green to orange, as 5 : 6, and purple to green, as 6 : 7.

These scales may therefore be termed scales for the colour-blind, because, in comparing colours, those whose sight is thus defective, naturally compare the ratios of the light and shade of which different colours are primarily constituted.



The following is a series of five complete scales of the harmonic parts into which the light and shade in colours may be divided in each scale according to the above arrangement:—

Second Series of Scales.

	Tonic.	Super-tonic.	Mediant.	Sub-dominant.	Dominant.	Sub-mediant.	Sub-tonic.	Semi-subtonic.	Tonic.
I.	$(\frac{1}{2})$ Yellow.	$(\frac{4}{9})$ Yellow-orange.	$(\frac{2}{5})$ Orange.	$(\frac{3}{8})$ Red-orange.	$(\frac{1}{3})$ Red.	$(\frac{3}{10})$ Red-purple.	$(\frac{2}{7})$ Purple.	$(\frac{4}{15})$ Blue-purple.	$(\frac{1}{4})$ Blue.
II.	$(\frac{1}{4})$ Purple.	$(\frac{2}{9})$ Blue-purple hue.	$(\frac{1}{5})$ Blue hue.	$(\frac{3}{16})$ Blue-green hue.	$(\frac{1}{6})$ Green.	$(\frac{3}{20})$ Yellow-green hue.	$(\frac{1}{7})$ Yellow hue.	$(\frac{2}{15})$ Yellow-orange hue.	$(\frac{1}{8})$ Orange.
III.	$(\frac{1}{8})$ Yellow hue.	$(\frac{1}{9})$ Yellow-orange hue.	$(\frac{1}{10})$ Orange hue.	$(\frac{3}{32})$ Red-orange hue.	$(\frac{1}{12})$ Red hue.	$(\frac{3}{40})$ Red-purple hue.	$(\frac{1}{14})$ Purple hue.	$(\frac{1}{15})$ Blue-purple hue.	$(\frac{1}{16})$ Blue hue.
IV.	$(\frac{1}{16})$ Purple hue.	$(\frac{1}{18})$ Blue-purple-toned gray.	$(\frac{1}{20})$ Blue-toned gray	$(\frac{3}{64})$ Blue-green-toned gray.	$(\frac{1}{24})$ Green hue.	$(\frac{3}{80})$ Yellow-green-toned gray.	$(\frac{1}{28})$ Yellow-toned gray.	$(\frac{1}{30})$ Yellow-orange-toned gray.	$(\frac{1}{32})$ Orange hue.
V.	$(\frac{1}{32})$ Yellow-toned gray.	$(\frac{1}{36})$ Yellow-orange-toned gray.	$(\frac{1}{40})$ Orange-toned gray.	$(\frac{3}{128})$ Red-orange-toned gray.	$(\frac{1}{48})$ Red-toned gray.	$(\frac{3}{160})$ Red-purple-toned gray.	$(\frac{1}{56})$ Purple-toned gray.	$(\frac{1}{60})$ Blue-purple-toned gray.	$(\frac{1}{64})$ Blue-toned gray.

Should I be correct in arranging colours upon scales identical with those upon which musical notes have been arranged, and in assuming that colours have the same ratios to each other, in respect to their harmonic power upon the eye, which musical notes have in respect to their harmonic power upon the ear, the colourist may yet be enabled to impart harmonic beauty to his works with as much certainty and ease, as the musician imparts the same quality to his compositions: for the colourist has no more right to trust exclusively to his eye in the arrangement of colours, than the musician has to trust exclusively to his ear in the arrangement of sounds.

We find, in comparing the dominant parts in the first and second scales of the second series, that they are equal as to light and shade, so that their relative powers of contrast depend entirely upon colour. Hence it is that red and green are the two colours, the difference between which the colour-blind are least able to appreciate. Professor George Wilson, in his excellent work, "Researches on Colour-Blindness," mentions the case of an engraver, which proves the power of the eye in being able to appreciate these original constituents of colour, irrespective of the intermediate phenomenon of tone. This engraver, instead of expressing regret on account of his being colour-blind, observed to the professor, "My defective vision is, to a certain extent, a useful and valuable quality. Thus, an engraver has two negatives to deal with, *i. e.*, white and black. Now, when I look at a picture, I see it only in white and black, or light and shade, or, as artists term it, the effect. I find at times many of my brother engravers in doubt how to translate certain colours of pictures, which to me are matters of decided certainty and ease. Thus to me it is valuable."

The colour-blind are therefore as incapable of receiving pleasure from the harmonious union of various colours, as those who, to use a common term, have no ear for music, are of being gratified by the "melody of sweet sounds."

The generality of mankind are, however, capable of appreciating the harmony of colour which, like that of both sound and form, arises from the simultaneous exhibition of opposite principles having a ratio to each other. These principles are in continual operation throughout nature, and from them we often derive pleasure without being conscious of the cause. All who are not colour-blind must have felt themselves struck with the harmonic beauty of a cloudless sky, although in it there is no configuration, and at first sight apparently but one colour. Now, as we know that there can be no more impression of harmony made upon the mind by looking upon a single colour, than there could be by listening to a single continued musical note, however sweet its tone, we are apt at first to imagine that the organ of vision has, in some measure, conveyed a false impression to the mind. But it has not done so; for light, when reflected from the atmosphere, produces those cool tones of blue, gray, and purple, which seem to clothe the distant mountains; but, when transmitted through the same atmosphere, it produces those numerous warm tints, the most intense of which give the gorgeous effects which so often accompany the setting sun. We have, therefore, in the upper part of a clear sky, where the atmosphere may be said to be illuminated principally by reflection from the surface of the earth, a comparatively cool tone of blue, the result of reflection, which gradually blends into the warm tints, the result of transmission through the same atmosphere. Such a composition of harmonious colouring is to the eye what the voice of the soft breath of summer amongst the trees, the hum of insects on a sultry day, or the simple harmony of the Æolian harp, is to the ear. To such a composition of chromatic harmony must also be referred the universal concurrence of mankind in appreciating the peculiar beauty of white marble statuary. That the principal constituent of beauty in such works ought to be harmony of form, no one will deny; but this is not the only element, as appears from the fact, that a cast in plaster of Paris, of a fine white marble statue, although identical in form, is far less beautiful than the original. Now this undoubtedly must be the consequence of its having been changed from a semi-translucent substance, which, like the atmosphere, can transmit as well as reflect light, to an opaque substance, which can only reflect it. Thus the opposite principles of chromatic warmth and coolness are equally balanced in white marble—the one being the natural result of the partial transmission of light, and the other that of its reflection.

As a series of coloured illustrations would be beyond the scope of this *résumé,* I may refer those who wish to prosecute the inquiry, with the assistance of such a series, to my published works upon the subject.*

* "The Principles of Beauty in Colouring Systematised," Fourteen Diagrams, each containing Six Colours and Hues.

"A Nomenclature of Colours," &c., Forty Diagrams, each containing Twelve Examples of Colours, Hues, Tints, and Shades.

"The Laws of Harmonious Colouring," &c., One Diagram, containing Eighteen Colours and Hues.

THE SCIENCE OF BEAUTY, APPLIED TO THE FORMS AND PROPORTIONS OF ANCIENT GRECIAN VASES AND ORNAMENTS.

In examining the remains of the ornamental works of the ancient Greek artists, it appears highly probable that the harmony of their proportions and melody of their contour are equally the result of a systematised application of the same harmonic law. This probability not being fully elucidated in any of my former works, I will require to go into some detail on the present occasion. I take for my first illustration an unexceptionable example, viz.:—

The Portland Vase.

Although this beautiful specimen of ancient art was found about the middle of the sixteenth century, inclosed in a marble sarcophagus within a sepulchral chamber under the Monte del Grano, near Rome, and although the date of its production is unknown, yet its being a work of ancient Grecian art is undoubted; and the exquisite beauty of its form has been universally acknowledged, both during the time it remained in the palace of the Barberini family at Rome, and since it was added to the treasures of the British Museum. The forms and proportions of this gem of art appear to me to yield an obedience to the great harmonic law of nature, similar to that which I have instanced in the proportions and contour of the best specimens of ancient Grecian architecture.

Let the line A B (Plate XII.) represent the full height of the vase. Through A draw A *a*, and through B draw B *b* indefinitely, A *a* making an angle of ($\frac{1}{2}$), and B *b* an angle of ($\frac{1}{3}$), with the vertical. Through the point C, where A *a* and B *b* intersect one another, draw D C E vertical. Through A C and B respectively, draw A D, C F, and B E horizontal. Draw similar lines on the other side of A B, and the rectilinear portion of the diagram is complete. Plate XII.

The curvilinear contour may be thus added:—

Take a cut-out ellipse of ($\frac{1}{4}$), whose greater axis is equal to the line A B, and

1*st*. Place it upon the diagram, so that its circumference may be tangential to the lines C E and C F, and its greater axis *m n* may make an angle of ($\frac{1}{5}$) with the vertical, and trace its circumference.

2*d*. Place it with its circumference tangential to that of the first at the point *m*, while its greater axis (of which *o p* is a part) is in the horizontal, and trace the portion of its circumference *q o r*.

3*d*. Place it with its circumference tangential to that of the above at *v*, while its greater axis (of which *u v* is a part) makes an angle of ($\frac{3}{10}$) with the vertical, and trace the portion of its circumference *s v t*.

Thus the curvilinear contour of the body and neck are harmonically determined.

The curve of the handle may be determined by the same ellipse placed so that its greater axis (of which *i k* is a part) makes an angle of ($\frac{1}{6}$) with the vertical.

Make similar tracings on the other side of A B, and the diagram is complete. The inscribing rectangle D G E K is that of ($\frac{2}{5}$).

The outline resulting from this diagram, not only is in perfect agreement with my recollection of the form, but with the measurements of the original given in the "Penny Cyclopædia;" of the accuracy of which there can be no doubt. They are stated thus:—"It is about ten inches in height, and beautifully curved from the top downwards; the diameter at the top being about three inches and a-half; at the neck or smallest part, two inches; at the largest (mid-height), seven inches; and at the bottom, five inches."

The harmonic elements of this beautiful form, therefore, appear to be the following parts of the right angle:—

Tonic.	Dominant.	Mediant.	Submediant.
($\frac{1}{2}$)	($\frac{1}{3}$)	($\frac{1}{5}$)	($\frac{3}{10}$)
($\frac{1}{4}$)	($\frac{1}{6}$)		

When we reflect upon the variety of harmonic ellipses that may be described, and the innumerable positions in which they may be harmonically placed with respect to the horizontal and vertical lines, as well as upon the various modes in which their circumferences may be combined, the variety which may be introduced amongst such forms as the foregoing appears almost endless. My second example is that of—

An Ancient Grecian Marble Vase of a Vertical Composition.

I shall now proceed to another class of the ancient Greek vase, the form of which is of a more complex character. The specimen I have chosen for the first example of this class is one of those so correctly measured and beautifully delineated by Tatham in his unequalled work.* This vase is a work of ancient Grecian art in Parian marble, which he met with in the collection at the Villa Albani, near Rome. Its height is 4 ft. $4\frac{1}{2}$ in.

The following is the formula by which I endeavour to develop its harmonic elements:—

Let A B (Plate XIII.) represent the full height of this vase. Plate XIII. Through B draw B D, making an angle of ($\frac{1}{5}$) with the vertical. Through D draw D O vertical, through A draw A C, making an angle of ($\frac{2}{5}$); through B draw B L, making an angle of ($\frac{1}{2}$), and B S, making an angle of ($\frac{3}{10}$), each with the vertical. Through A draw A D, through B draw B O, through L draw L N, through C draw C F, and through S draw S P, all horizontal. Through A draw A H, making an angle of ($\frac{1}{10}$) with the vertical, and through H draw H M vertical. Draw similar lines on the other side of A B, and the rectilinear portion of the diagram is complete, and its inscribing rectangle that of ($\frac{3}{8}$).

The curvilinear portion may thus be added—

Take a cut-out ellipse of ($\frac{1}{3}$), whose greater axis is about the length of the body of the intended vase, place it with its lesser axis upon the line S P, and its greater axis upon the line D O, and trace the part *a b* of its circumference upon the diagram. Place the same ellipse with one of its foci upon C, and its greater axis upon C F, and trace its circumference upon the diagram. Take a cut-out ellipse of ($\frac{1}{5}$), whose greater axis is nearly equal to that of the ellipse already used;

* "Etchings Representing the Best Examples of Grecian and Roman Architectural Ornament, drawn from the Originals," &c. By Charles Heathcote Tatham, Architect. London: Priestly and Weale. 1826.

place it with its greater axis upon M H, and its lesser axis upon L N, and trace its circumference upon the diagram. Make similar tracings upon the other side of A B, and the diagram is complete. In this, as in the other diagrams, the strong portions of the lines give the contour of the vase. The harmonic elements of this classical form, therefore, appear to be the right angle and its following parts:—

Tonic.	Dominant.	Mediant.	Submediant.
$(\frac{1}{2})$	$(\frac{1}{3})$	$(\frac{2}{5})$	$(\frac{3}{10})$
		$(\frac{1}{5})$	
		$(\frac{1}{10})$	

My third example is that of—

An Ancient Grecian Vase of a Horizontal Composition.

This example belongs to the same class as the last, but it is of a horizontal composition. It was carefully drawn from the original in the museum of the Vatican by Tatham, in whose etchings it will be found with its ornamental decorations. The diagram of its harmonic elements may be constructed as follows:—

Plate XIV. Let A B (Plate XIV.) represent the full height of the vase. Through B draw B D, making an angle of $(\frac{2}{5})$ with the vertical. Through A draw A H, A L, and A C, making respectively the following angles, $(\frac{1}{5})$ with the vertical, $(\frac{4}{9})$ with the vertical, and $(\frac{3}{10})$ with the horizontal. These angles determine the horizontal lines H B, L N, and C F, which divide the vase into its parts, and the inscribing rectangle D G K O is $(\frac{3}{8})$. This completes the rectilinear portion of the diagram. The ellipse by which the curvilinear portion is added is one of $(\frac{1}{5})$, the greater axis of which, at *a b*, as also at *c d*, makes an angle of $(\frac{1}{12})$ with the vertical, and the same axis at *e f* an angle of $(\frac{1}{12})$ with the horizontal.

The harmonic elements of this vase, therefore, appear to be:—

Tonic.	Dominant.	Mediant.	Submediant.	Supertonic.
The Right Angle.	$(\frac{1}{12})$	$(\frac{2}{5})$	$(\frac{3}{10})$	$(\frac{4}{9})$
		$(\frac{1}{5})$		

My remaining examples are those of—

Etruscan Vases.

Of these vases I give four examples, by which the simplicity of the method employed in applying the harmonic law will be apparent.

The inscribing rectangle D G E K of fig. 1, Plate XV., is one of $(\frac{3}{8})$, within which are arranged tracings from an ellipse of $(\frac{3}{10})$, whose greater axis at *a b* makes an angle of $(\frac{1}{12})$, at *c d* an angle of $(\frac{3}{10})$, and at *e f* an angle of $(\frac{3}{4})$, with the vertical. The harmonic elements of the contour of this vase, therefore, appear to be:— Plate XV.

Tonic.	Dominant.	Subdominants.	Submediant.
The Right Angle.	$(\frac{1}{12})$	$(\frac{3}{4})$	$(\frac{3}{10})$
		$(\frac{3}{8})$	

The inscribing rectangle L M N O of fig. 2 is that of $(\frac{1}{2})$, within which are arranged tracings from an ellipse of $(\frac{1}{3})$, whose greater axis, at *a b* and *c d* respectively, makes angles of $(\frac{1}{2})$ and $(\frac{4}{9})$ with the horizontal, while that at *e f* is in the horizontal line. The harmonic elements of the contour of this vase, therefore, appear to be:—

Tonic.	Dominant.	Subtonic.
$(\frac{1}{2})$	$(\frac{1}{3})$	$(\frac{4}{9})$

Plate XVI. The inscribing rectangle P Q R S of fig. 1, Plate XVI., is one of $(\frac{4}{9})$, within which are arranged tracings from an ellipse of $(\frac{3}{8})$, whose greater axis, at *a b*, *c d*, and *e f*, makes respectively angles of $(\frac{1}{6})$ with the horizontal, $(\frac{3}{5})$ and $(\frac{4}{5})$ with the vertical. Its harmonic elements, therefore, appear to be:—

Tonic.	Dominant.	Mediant.	Supertonic.	Subdominant.	Submediant.
The Right Angle.	$(\frac{1}{6})$	$(\frac{4}{5})$	$(\frac{4}{9})$	$(\frac{3}{8})$	$(\frac{3}{5})$

The inscribing rectangle T U V X of fig. 2 is one of $(\frac{4}{9})$, within which are arranged tracings from an ellipse of $(\frac{3}{8})$ whose greater axis at *a b* is in the vertical line, and at *c d* makes an angle of $(\frac{1}{2})$. The harmonic elements of the contour of this vase, therefore, appear to be:—

Tonic.	Submediant.	Supertonic.
$(\frac{1}{2})$	$(\frac{3}{8})$	$(\frac{4}{9})$

These four Etruscan vases, the contours of which are thus reduced to the harmonic law of nature, are in the British Museum, and engravings of them are to be found in the well-known work of Mr Henry Moses, Plates 4, 6, 14, and 7, respectively, where they are represented with their appropriate decorations and colours.

To these, I add two examples of—

Ancient Grecian Ornament.

I have elsewhere shewn * that the elliptic curve pervades the Parthenon from the entases of the column to the smallest moulding, and we need not, therefore, be surprised to find it

* "The Orthographic Beauty of the Parthenon," &c.

employed in the construction of the only two ornaments belonging to that great work.

In the diagram (Plate XVII.), I endeavour to exhibit the geometric construction of the upper part of one of the ornamental apices, termed antefixæ, which surmounted the cornice of the Parthenon. Plate XVII.

The first ellipse employed is that of ($\frac{1}{3}$), whose greater axis *a b* is in the vertical line; the second is also that of ($\frac{1}{3}$), whose greater axis *c d* makes, with the vertical, an angle of ($\frac{1}{12}$); the third ellipse is the same with its major axis *e f* in the vertical line. Through one of the foci of this ellipse at A the line A C is drawn, and upon the part of the circumference C *e*, the number of parts, 1, 2, 3, 4, 5, 6, 7, of which the surmounting part of this ornament is to consist, are set off. That part of the circumference of the ellipse whose larger axis is *c d* is divided from *g* to *c* into a like number of parts. The third ellipse employed is one of ($\frac{1}{4}$).

Take a cut-out ellipse of this kind, whose larger axis is equal in length to the inscribing rectangle. Place it with its vertex upon the same ellipse at *g*, so that its circumference will pass through C, and trace it; remove its apix first to *p*, then to *q*, and proceed in the same way to *q*, *r*, *s*, *t*, *u*, and *v*, so that its circumference will pass through the seven divisions on *c g* and *e* C: *v o*, *u n*, *t m*, *s i*, *r k*, *q j*, *p l*, and *g x*, are parts of the larger axes of the ellipses from which the curves are traced. The small ellipse of which the ends of the parts are formed is that of ($\frac{1}{3}$).

In the diagram (Plate XVIII.), I endeavour to exhibit the geometric construction of the ancient Grecian ornament, commonly called the *Honeysuckle*, from its resemblance to the flower of that name. The first part of the process is similar to that just explained with reference to the antefixæ of the Parthe- Plate XVIII.

non, although the angles in some parts differ. The contour is determined by the circumference of an ellipse of ($\frac{1}{3}$), whose major axis A B makes an angle of ($\frac{1}{9}$) with the vertical, and the leaves or petals are arranged upon a portion of the perimeter of a similar ellipse whose larger axis E F is in the vertical line, and these parts are again arranged upon a similar ellipse whose larger axis C D makes an angle of ($\frac{1}{12}$) with the vertical. The first series of curved lines proceeding from 1, 2, 3, 4, 5, 6, 7, and 8, are between K E and H C, part of the circumference of an ellipse of ($\frac{1}{3}$); and those between C H and A G are parts of the circumference of four ellipses, each of ($\frac{1}{3}$), but varying as to the lengths of their larger axes from 5 to 3 inches. The change from the convex to the concave, which produces the ogie forms of which this ornament is composed, takes place upon the line C H, and the lines *a b, c d, e f, g h, i k, l m, n o,* and *p q,* are parts of the larger axis of the four ellipses the circumference of which give the upper parts of the petals or leaves.

This peculiar Grecian ornament is often, like the antefixæ of the Parthenon, combined with the curve of the spiral scroll. But the volute is so well understood that I have not rendered my diagrams more complex by adding that figure. Many varieties of this union are to be found in Tatham's etchings, already referred to. The antefixæ of the Parthenon, and its only other ornament the honeysuckle, as represented on the soffit of the cornice, are to be found in Stewart's "Athens."

APPENDIX.

No. I.

In pages 34, 35, and 58, I have reiterated an opinion advanced in several of my former works, viz., that, besides genius, and the study of nature, an additional cause must be assigned for the general excellence which characterises such works of Grecian art as were executed during a period commencing about 500 B. C., and ending about 200 B. C. And that this cause most probably was, that the artists of that period were instructed in a system of fixed principles, based upon the doctrinces of Pythagoras and Plato. This opinion has not been objected to by the generality of those critics who have reviewed my works; but has, however, met with an opponent, whose recondite researches and learned observations are worthy of particular attention. These are given in an essay by Mr C. Knight Watson, "On the Classical Authorities for Ancient Art," which appeared in the *Cambridge Journal of Classical and Sacred Philology* in June 1854. As this essay is not otherwise likely to meet the eyes of the generality of my readers, and as the objections he raises to my opinion only occupy two out of the sixteen ample paragraphs which constitute the first part of the essay, I shall quote them fully :—

"The next name on our list is that of the famous Euphranor (B. C. 362). For the fact that to the practice of sculpture and of painting he added an exposition of the theory, we are indebted to Pliny, who says (xxxv. 11, 40), 'Volumina quoque composuit de symmetria et coloribus.' When we reflect on the *critical* position occupied by Euphranor in the history of Greek art, as a connecting link between the idealism of Pheidias and the naturalism of Lysippus, we can scarcely overestimate the value of a treatise on art proceeding from such a quarter. This is especially the case with the first of the two works here assigned to Euphranor. The inquiries which of late years have been instituted by Mr D. R. Hay of Edinburgh, on the proportions of the human figure, and on the natural principles of beauty as illustrated by works of Greek art, constitute an epoch in the study of æsthetics and the philosophy of form. Now, in the presence of these inquiries, or of such less solid results as Mr Hay's predecessors in the same field have elicited, it naturally becomes an object of considerable interest to ascertain how far these laws of form and principles of beauty were consciously developed in the mind, and by the chisel, of the sculptor: how far any such system of curves and proportions as Mr Hay's was used by the Greek as a practical manual of his craft. Without in the least wishing to impugn the accuracy of that gentleman's results—a piece of presumption I should do well to avoid—I must be permitted to doubt whether the 'Symmetria' of Euphranor contained anything analogous to them in kind, or indeed equal in value. It must not be forgotten that the truth of Mr Hay's theory is perfectly compatible with the fact, that of such theory the Greek may have been utterly ignorant. It is on this fact I insist: it is here that I join issue with Mr Hay, and with his reviewer in a recent number of *Blackwood's Magazine.* Or, to speak more accurately,—while I am quite prepared to find that the Elgin marbles will best of all stand the test which Mr Hay has hitherto applied, I believe, to works of a later age, I am none the less convinced that it is precisely that golden age of Hellenic art to which they belong, precisely that first and chief of Hellenic artists by whom they were executed, to which and to whom any such line of research on the laws of form would have been pre-eminently alien. Pheidias, remember, by the right of primogeniture, is the ruling spirit of idealism in art. Of spontaneity was that idealism begotten and nurtured: by any such system as Mr Hay's, that spontaneity would be smothered and paralysed. Pheidias copied an idea in his own mind—'Ipsius in mente insidebat species pulchritudinis eximia quædam' (*Cic.*);—later ages copied *him.* He created: they criticised. He was the author of Iliads: they the authors of Poetics. Doubtless, if you unsphere the *spirit* of Mr Hay's theories, you will find nothing discordant with what I have here said. That is a sound view of Beauty which makes it consist in that due subordination of the parts to the whole, that due relation of the parts to each other, which Mendelssohn had in his mind when he said that the essence of beauty was 'unity in variety'—variety beguiling the imagination, the perception of unity exercising the thewes and sinews of the intellect. On such a view of beauty, Mr Hay's theory may, *in spirit,* be said to rest. But here, as in higher things, it is the letter that killeth, while the spirit giveth life. And accordingly I must enter a protest against any endeavour to foist upon the palmy days of Hellenic art systems of geometrical proportions incompatible, as I believe, with those higher and broader principles by which the progress of ancient sculpture was ordered and governed—systems which will bear nothing of that 'felicity and chance by which'—and not by rule—'Lord Bacon believed that a painter may make a better face than ever was:' systems which take no account of that fundamental distinction between the schools of

Athens and of Argos, and their respective disciples and descendants, without which you will make nonsense of the pages of Pliny, and—what is worse—sense of the pages of his commentators ;—systems, in short, which may have their value as instruments for the education of the eye, and for instructions in the arts of design, but must be cast aside as matters of learned trifling and curious disputation, where they profess to be royal roads to art, and to map the mighty maze of a creative mind. And even as regards the application of such a system of proportions to those works of sculpture which are posterior to the Pheidian age, only partial can have been the prevalence which it or any other *one* system can have obtained. The discrepancies of different artists in the treatment of what was called, technically called, *Symmetria* (as in the title of Euphranor's work) were, by the concurrent testimony of all ancient writers, far too salient and important to warrant the supposition of any uniform scale of proportions, as advocated by Mr Hay. Even in Egypt, where one might surely have expected that such uniformity would have been observed with far greater rigour than in Greece, the discoveries of Dr Lepsius (*Vorläufige Nachricht*, Berlin, 1849) have elicited three totally different κανόνες, one of which is identical with the system of proportions of the human figure detailed in Diodorus. While we thus venture to differ from Mr Hay on the historical data he has mixed up with his inquiries, we feel bound to pay him a large and glad tribute of praise for having devised a system of proportions which rises superior to the idiosyncracies of different artists, which brings back to one common type the sensations of eye and ear, and so makes a giant stride towards that *codification*, if I may so speak, of the laws of the universe which it is the business of the science to effect. I have no hesitation in saying, that, for scientific precision of method and importance of results, Albert Durer, Da Vinci, and Hogarth, not to mention less noteworthy writers, must all yield the palm to Mr Hay.

"I am quite aware that in the digression I have here allowed myself, on systems of proportions prevalent among ancient artists, and on the probable contents of such treatises as that of Euphranor, *De Symmetria*, I have laid myself open to the charge of treating an intricate question in a very perfunctory way. At present the exigencies of the subject more immediately in hand allow me only to urge in reply, that, as regards the point at issue—I mean the 'solidarité' between theories such as Mr Hay's and the practice of Pheidias—the *onus probandi* rests with my adversaries."

I am bound, in the first place, gratefully to acknowledge the kind and complimentary notice which, notwithstanding our difference of opinion, this author has been pleased to take of my works ; and, in the second, to assure him that if any of them profess to be "royal roads to art," or to "map the mighty maze of a creative mind," they certainly profess to do more than I ever meant they should ; for I never entertained the idea that a system of æsthetic culture, even when based upon a law of nature, was capable of effecting any such object. But I doubt not that this too common misapprehension of the scope and tendency of my works must arise from a want of perspicuity in my style.

I have certainly, on one occasion,* gone the length of stating that as poetic genius must yield obedience to the rules of rythmical measure, even in the highest flights of her inspirations ; and musical genius must, in like manner, be subject to the strictly defined laws of harmony in the most delicate, as well as in the most powerfully grand of her compositions ; so must genius, in the formative arts, either consciously or unconsciously have clothed her creations of ideal beauty with proportions strictly in accordance with the laws which nature has set up as inflexible standards. If, therefore, the laws of proportion, in their relation to the arts of design, constitute the harmony of geometry, as definitely as those that are applicable to poetry and music produce the harmony of acoustics ; the former ought, certainly, to hold the same relative position in those arts which are addressed to the eye, that is accorded to the latter in those which are addressed to the ear. Until so much science be brought to bear upon the arts of design, the student must continue to copy from individual and imperfect objects in nature, or from the few existing remains of ancient Greek art, in total ignorance of the laws by which their proportions are produced, and, what is equally detrimental to art, the accuracy of all criticism must continue to rest upon the indefinite and variable basis of mere opinion.

It cannot be denied that men of great artistic genius are possessed of an intuitive feeling of appreciation for what is beautiful, by means of which they impart to their works the most perfect proportions, independently of any knowledge of the definite laws which govern that species of beauty. But they often do so at the expense of much labour, making many trials before they can satisfy themselves in imparting to them the true proportions which their minds can conceive, and which, along with those other qualities of expression, action, or attitude, which belong more exclusively to the province of genius. In such cases, an acquaintance with the rules which constitute the science of proportion, instead of proving fetters to genius, would doubtless afford her such a vantage ground as would promote the more free exercise of her powers, and give confidence and precision in the embodiment of her inspirations; qualities which, although quite compatible with genius, are not always intuitively developed along with that gift.

It is also true that the operations of the conceptive faculty of the mind are uncontrolled by definite laws, and that, therefore, there cannot exist any rules by the inculcation of which an ordinary mind can be imbued with genius sufficient to produce works of high art. Nevertheless, such a mind may be improved in its perceptive faculty by instruction in the science of proportion, so as to be enabled to exercise as correct and just an appreciation of

* " Science of those Proportions," &c.

the conceptions of others, in works of plastic art, as that manifested by the educated portion of mankind in respect to poetry and music. In short, it appears that, in those arts which are addressed to the ear, men of genius communicate the original conceptions of their minds under the control of certain scientific laws, by means of which the educated easily distinguish the true from the false, and by which the works of the poet and musical composer may be placed above mere imitations of nature, or of the works of others; while, in those arts that are addressed to the eye in their own peculiar language, such as sculpture, architecture, painting, and ornamental design, no such laws are as yet acknowledged.

Although I am, and ever have been, far from endeavouring "to foist upon the palmy days of Hellenic art" any system incompatible with those higher and more intellectual qualities which genius alone can impart; yet, from what has been handed down to us by writers on the subject, meagre as it is, I cannot help continuing to believe that, along with the physical and metaphysical sciences, æsthetic science was taught in the early schools of Greece.

I shall here take the liberty to repeat the proofs I advanced in a former work as the ground of this belief, and to which the author, from whose essay I have quoted, undoubtedly refers. It is well known that, in the time of Pythagoras, the treasures of science were veiled in mystery to all but the properly initiated, and the results of its various branches only given to the world in the works of those who had acquired this knowledge. So strictly was this secresy maintained amongst the disciples and pupils of Pythagoras, that any one divulging the sacred doctrines to the profane, was expelled the community, and none of his former associates allowed to hold further intercourse with him; it is even said, that one of his pupils incurred the displeasure of the philosopher for having published the solution of a problem in geometry.* The difficulty, therefore, which is expressed by writers, shortly after the period in which Pythagoras lived, regarding a precise knowledge of his theories, is not to be wondered at, more especially when it is considered that he never committed them to writing. It would appear, however, that he proceeded upon the principle, that the order and beauty so apparent throughout the whole universe, must compel men to believe in, and refer them to, an intelligible cause. Pythagoras and his disciples sought for properties in the science of numbers, by the knowledge of which they might attain to that of nature; and they conceived those properties to be indicated in the phenomena of sonorous bodies. Observing that Nature herself had thus irrevocably fixed the numerical value of the intervals of musical tones, they justly concluded that, as she is always uniform in her works, the same laws must regulate the general system of the universe.†

* Abbé Barthélémie's "Travels of Anacharsis in Greece," vol iv., pp. 193, 195.

† Abbé Barthélémie (vol. ii., pp. 168, 169), who cites as his authorities, Cicer. De Nat. Deor., lib. i., cap. ii., t. 2, p. 405; Justin Mart., Ovat. ad Gent., p. 10; Aristot. Metaph., lib. i., cap. v., t. 2, p. 845.

Pythagoras, therefore, considered numerical proportion as the great principle inherent in all things, and traced the various forms and phenomena of the world to numbers as their basis and essence.

How the principles of numbers were applied in the arts is not recorded, farther than what transpires in the works of Plato, whose doctrines were from the school of Pythagoras. In explaining the principle of beauty, as developed in the elements of the material world, he commences in the following words :—"But when the Artificer began to adorn the universe, he first of all figured with forms and numbers, fire and earth, water and air—which possessed, indeed, certain traces of the true elements, but were in every respect so constituted as it becomes anything to be from which Deity is absent. But we should always persevere in asserting that Divinity rendered them, as much as possible, the most beautiful and the best, when they were in a state of existence opposite to such a condition." Plato goes on further to say, that these elementary bodies must have forms; and as it is necessary that every depth should comprehend the nature of a plane, and as of plane figures the triangle is the most elementary, he adopts two triangles as the originals or representatives of the isosceles and the scalene kinds. The first triangle of Plato is that which forms the half of the square, and is regulated by the number, 2; and the second, that which forms the half of the equilateral triangle, which is regulated by the number, 3; from various combinations of these, he formed the bodies of which he considered the elements to be composed. To these elementary figures I have already referred.

Vitruvius, who studied architecture ages after the arts of Greece had been buried in the oblivion which succeeded her conquest, gives the measurements of various details of monuments of Greek art then existing. But he seems to have had but a vague traditionary knowledge of the principle of harmony and proportion from which these measurements resulted. He says —"The several parts which constitute a temple ought to be subject to the laws of symmetry; the principles of which should be familiar to all who profess the science of architecture. Symmetry results from proportion, which, in the Greek language, is termed analogy. Proportion is the commensuration of the various constituent parts with the whole; in the existence of which symmetry is found to consist. For no building can possess the attributes of composition in which symmetry and proportion are disregarded; nor unless there exist that perfect conformation of parts which may be observed in a well-formed human being." After going at some length into details, he adds—"Since, therefore, the human figure appears to have been formed with such propriety, that the several members are commensurate with the whole, the artists of antiquity (meaning those of Greece at the period of her highest refinement) must be allowed to have followed the dictates of a judgment the most rational, when, transferring to works of art principles derived from nature, every part was so regulated as to bear a just proportion to the whole. Now, although the principles were universally acted upon, yet they were more particularly attended to in the construction of temples and sacred edifices, the beauties or defects of which were destined to remain as a perpetual testimony of their skill or of their inability."

Vitruvius, however, gives no explanation of this ancient principle of proportion, as derived from the human form; but plainly shews his uncertainty upon the subject, by concluding this part of his essay in the following words: "If it be true, therefore, that the decenary notation was suggested by the members of man, and that the laws of proportion arose from the relative measures existing between certain parts of each member and the whole body, it will follow, that those are entitled to our commendation who, in building temples to their deities, proportioned the edifices, so that the several parts of them might be commensurate with the whole."

It thus appears certain that the Grecians, at the period of their highest excellence, had arrived at a knowledge of some definite mathematical law of proportion, which formed a standard of perfectly symmetrical beauty, not only in the representation of the human figure in sculpture and painting, but in architectural design, and indeed in all works where beauty of form and harmony of proportion constituted excellence. That this law was not deduced from the proportions of the human figure, as supposed by Vitruvius, but had its origin in mathematical science, seems equally certain; for in no other way can we satisfactorily account for the proportions of the beau ideal forms of the ancient Greek deities, or of those of their architectural structures, such as the Parthenon, the temple of Theseus, &c., or for the beauty that pervades all the other formative art of the period.

This system of geometrical harmony, founded, as I have shewn it to be, upon numerical relations, must consequently have formed part of the Greek philosophy of the period, by means of which the arts began to progress towards that great excellence which they soon after attained. A little further investigation will shew, that immediately after this period a theory connected with art was acknowledged and taught, and also that there existed a Science of Proportion.

Pamphilus, the celebrated painter, who flourished about four hundred years before the Christian era, from whom Apelles received the rudiments of his art, and whose school was distinguished for scientific cultivation, artistic knowledge, and the greatest accuracy in drawing, would admit no pupil unacquainted with geometry.* The terms upon which he engaged with his students were, that each should pay him one talent (£225 sterling) previous to receiving his instructions; for this he engaged "to give them, *for ten years*, lessons founded on an excellent theory." †

It was by the advice of Pamphilus that the magistrates of Sicyon ordained that the study of drawing should constitute part of the education of the

* Müller's "Ancient Art and its Remains."

† "Anacharsis' Travels in Greece." By the Abbé Barthélémie, vol. ii., p. 325.

citizens—"a law," says the Abbé Barthélemie, "which rescued the fine arts from servile hands."

It is stated of Parrhasius, the rival of Zeuxis, who flourished about the same period as Pamphilus, that he accelerated the progress of art by purity and correctness of design; "for he was acquainted with the science of Proportions. Those he gave his gods and heroes were so happy, that artists did not hesitate to adopt them." Parrhasius, it is also stated, was so admired by his contemporaries, that they decreed him the name of Legislator.* The whole history of the arts in Egypt and Greece concurs to prove that they were based on geometric precision, and were perfected by a continued application of the same science; while in all other countries we find them originating in rude and misshapen imitations of nature.

In the earliest stages of Greek art, the gods—then the only statues—were represented in a tranquil and fixed posture, with the features exhibiting a stiff inflexible earnestness, their only claim to excellence being symmetrical proportion; and this attention to geometric precision continued as art advanced towards its culminating point, and was thereafter still exhibited in the neatly and regularly folded drapery, and in the curiously braided and symmetrically arranged hair.†

These researches, imperfect as they are, cannot fail to exhibit the great contrast that exists between the system of elementary education in art practised in ancient Greece, and that adopted in this country at the present period. But it would be of very little service to point out this contrast, were it not accompanied by some attempt to develop the principles which seem to have formed the basis of this excellence amongst the Greeks.

But in making such an attempt, I cannot accuse myself of assuming anything incompatible with the free exercise of that spontaneity of genius which the learned essayist says is the parent and nurse of idealism. For it is in no way more incompatible with the free exercise of artistic genius, that those who are so gifted should have the advantage of an elementary education in the science of æsthetics, than it is incompatible with the free exercise of literary or poetic genius, that those who possess it should have the advantage of such an elementary education in the science of philology as our literary schools and colleges so amply afford.

* "Anacharsis' Travels in Greece." By the Abbé Barthélémie, vol. vi., p. 225. The authorities the Abbé quotes are—Quintil., lib. xii., cap. x., p. 744; Plin., lib. xxxv., cap. ix., p. 691.

† Müller's "Archæology of Art," &c.

No. II.

The letter from which I have made a quotation at page 42, arose out of the following circumstance:—In order that my theory, as applied to the orthographic beauty of the Parthenon, might be brought before the highest tribunal which this country afforded, I sent a paper upon the subject, accompanied by ample illustrations, to the Royal Institute of British Architects, and it was read at a meeting of that learned body on the 7th of February 1853; at the conclusion of which, Mr Penrose kindly undertook to examine my theoretical views, in connexion with the measurements he had taken of that beautiful structure by order of the Dilletanti Society, and report upon the subject to the Royal Institute. This report was published by Mr Penrose, vol. xi., No. 539 of *The Builder*, and the letter from which I have quoted appeared in No. 542 of the same journal. It was as follows:—

"GEOMETRICAL RELATIONS IN ARCHITECTURE.

"Will you allow me, through the medium of your columns, to thank Mr Penrose for his testimony to the truth of Mr Hay's revival of Pythagoras? The dimensions which he gives are to me the surest verification of the theory that I could have desired. The minute discrepancies form that very element of practical incertitude, both as to execution and direct measurement, which always prevails in materialising a mathematical calculation under such conditions.

"It is time that the scattered computations by which architecture has been analysed—more than enough—be synthetised into those formulæ which, as Mrs Somerville tells us, 'are emblematic of omniscience.' The young architects of our day feel trembling beneath their feet the ground whence men are about to evoke the great and slumbering corpse of art. Sir, it is food of this kind a reviving poetry demands.

———'Give us truths,
For we are weary of the surfaces,
And die of inanition.'

"I, for one, as I listen to such demonstrations, whose scope extends with every research into them, feel as if listening to those words of Pythagoras, which sowed in the mind of Greece the poetry whose manifestation in beauty has enchained the world in worship ever since its birth. And I am sure that in such a quarter, and in such thoughts, *we* must look for the first shining of that lamp of art, which even now is prepared to burn.

"I know that this all sounds rhapsodical; but I know also that until the architect becomes a poet, and not a tradesman, we may look in vain for architecture: and I know that valuable as isolated and detailed investigations are in their proper bearings, yet that such purposes and bearings are to be found in the enunciation of principles sublime as the generalities of 'mathematical beauty.' "AUTOCTHON."

No. III.

Of the work alluded to at page 58 I was favoured with two opinions—the one referring to the theory it propounds, and the other to its anatomical accuracy—both of which I have been kindly permitted to publish.

The first is from Sir WILLIAM HAMILTON, Bart., professor of logic and metaphysics in the University of Edinburgh, and is as follows:—

"Your very elegant volume is to me extremely interesting, as affording an able contribution to what is the ancient, and, I conceive, the true theory of the Beautiful. But though your doctrine coincides with the one prevalent through all antiquity, it appears to me quite independent and original in you; and I esteem it the more, that it stands opposed to the hundred one-sided and exclusive views prevalent in modern times.—16 *Great King Street, March* 5, 1849."

The second is from JOHN GOODSIR, Esq., professor of anatomy in the University of Edinburgh, and is as follows:—

"I have examined the plates in your work on the proportions of the human head and countenance, and find the head you have given as typical of human beauty to be anatomically correct in its structure, only differing from ordinary nature in its proportions being more mathematically precise, and consequently more symmetrically beautiful.—*College, Edinburgh*, 17*th April* 1849."

No. IV.

I shall here shew, as I have done in a former work, how the curvilinear outline of the figure is traced upon the rectilinear diagrams by portions of the ellipse of ($\frac{1}{3}$), ($\frac{1}{4}$), ($\frac{1}{5}$), and ($\frac{1}{6}$).

Plate XIX. The outline of the head and face, from points (1) to (3) (fig. 1, Plate XIX.), takes the direction of the two first curves of the diagram. From point (3), the outline of the sterno-mastoid muscle continues to (4), where, joining the outline of the trapezius muscle, at first concave, it becomes convex after passing through (5), reaches the point (6), where the convex outline of the deltoid muscle commences, and, passing through (7), takes the outline of the arm as far as (8). The outline of the muscles on the side, the latissimus dorsi and serratus magnus, commences under the arm at the point (9), and joins the outline of the oblique muscle of the abdomen by a concave curve at (10), which, rising into convexity as it passes through the points (11) and (12), ends at (13), where it joins the outline of the gluteus medius muscle. The outline of this latter muscle passes convexly through the point (14), and ends at (15), where the outline of the tensor vaginæ femoris and vastus externus muscle of the thigh commences. This convex outline joins the concave outline of the biceps of the thigh at (16), which ends in that of the slight convexity of the condyles of the thigh-bone at (17). From this point, the outline of the outer surface of the leg, which includes the biceps, peroneus longus, and soleus muscles, after passing through the point (18), continues convexly to (19), where the concave outline of the tendons of the peroneus longus continues to (20), whence the outline of the outer ankle and foot commences.

The outline of the mamma and fold of the arm-pit commences at (21), and passes through the points (22) and (23). The circle at (24) is the outline of the areola, in the centre of which the nipple is placed.

The outline of the pubes commences at (25), and ends at the point (26), from which the outline of the inner surface of the thigh proceeds over the gracilis, the sartorius, and vastus internus muscles, until it meets the internal face of the knee-joint at (27), the outline of which ends at (28). The outline of the inside of the leg commences by proceeding over the gastrocnemius muscle as far as (29), where it meets that of the soleus muscle, and continues over the tendons of the heel until it meets the outline of the inner ankle and foot at (30).

The outline of the outer surface of the arm, as viewed in front, proceeds from (8) over the remainder of the deltoid, in which there is here a slight concavity, and, next, from (31) over the biceps muscle till (32), where it takes the line of the long supinator, and passing concavely, and almost imperceptibly, into the long and short radial extensor muscles, reaches the wrist at (33). The outline of the inner surface of the arm from opposite (9) commences by passing over the triceps extensor, which ends at (34), then over the gentle convexity of the condyles of the bone of the arm at (35), and, lastly, over the flexor sublimis which ends at the wrist-joint (36).

The outline of the front of the figure commences at the point (1), (fig. 2, Plate II.), and, passing almost vertically over the platzsma-myoidis muscles, reaches the point (2), where the neck ends by a concave curve. From (2) the outline rises convexly over the ends of the clavicles, and continues so over the pectoral muscle till it reaches (3), where the mamma swells out convexly to (4), and returns convexly towards (5), where the curve becomes concave. From (5) the outline follows the undulations of the rectus muscle of the abdomen, concave at the points (6) and (7), and having its greatest convexity at (8). This series of curves ends with a slight concavity at the point (9), where the horizontal branch of the pubes is situated, over which the outline is convex and ends at (10).

The outline of the thigh commences at the point (11) with a slight concave curve, and then swells out convexly over the extensors of the leg, and, reaching (12), becomes gently concave, and, passing over the patella at (13), becomes again convex until it reaches the ligament of that bone, where it becomes gently concave towards the point (14), whence it follows the slightly convex curve of the shin-bone, and then, becoming as slightly concave, ends with the muscles in front of the leg at (15).

The outline of the back commences at the point (16), and, following with a concave curve the muscles of the neck as far as (17), swells into a convex curve over the trapezius muscle towards the point (18); passing through which point, it continues to swell outward until it reaches (19), half way between (18) and (20); whence the convexity, becoming less and less, falls into the concave curve of the muscles of the loins at (21), and passing through the point (22), it rises into convexity. It then passes through the point (23), follows the outline of the gluteus maximus, the convex curve of which rises to the point (24), and then returns inwards to that of (25), where it ends in the fold of the hip. From this point the outline follows the curve of the hamstring muscles by a slight concavity as far as (26), and then, becoming gently convex, it reaches (27); whence it becomes again gently concave, with a slight indication of the condyle of the thigh-bone at (28), and, reaching (29), follows the convex curve of the gastrocnemius muscle through the point (30), and falling into the convex curve of the tendo Achilles at (31), ends in the concavity over the heel at (32).

The outline of the front of the arm commences at the point (33), by a gentle concavity at the arm-pit, and then swells out in a convex curve over the biceps, reaching (34), where it becomes concave, and passing through (35), again becomes convex in passing over the long supinator, and, becoming gently concave as it passes the radial extensors, rises slightly at (36), and ends at (37), where the outline of the wrist commences. The outline of the back of the arm commences with a concave curve at (38), which becomes convex as it passes from the deltoid to the long extensor and ends at the elbow (39), from below which the outline follows the convex curve of the extensor ulnaris, reaching the wrist at the point (40).

It will be seen that the various undulations of the outline are regulated by points which are determined generally by the intersections and sometimes by directions and extensions of the lines of the diagram, in the same manner in which I shewed proportion to be imparted, in a late work, to the osseous structure. The mode in which the curves of ($\frac{1}{2}$), ($\frac{1}{3}$), ($\frac{1}{4}$), ($\frac{1}{5}$), and

($\frac{1}{6}$) are thus so harmoniously blended in the outline of the female figure, only remains to be explained.

The curves which compose the outline of the female form are therefore simply those of ($\frac{1}{2}$), ($\frac{1}{3}$), ($\frac{1}{4}$), ($\frac{1}{5}$), and ($\frac{1}{6}$).

Manner in which these curves are disposed in the lateral outline (figure 1, Plate XIX.):—

	Points.				Curves.
Head	from	1	to	2	($\frac{1}{2}$)
Face	„	2	„	3	($\frac{1}{3}$)
Neck	„	3	„	4	($\frac{1}{5}$)
Shoulder	„	4	„	6	($\frac{1}{6}$)
„	„	6	„	8	($\frac{1}{4}$)
Trunk	„	9	„	15	($\frac{1}{4}$)
„	„	21	„	24	($\frac{1}{2}$)
Outer surface of thigh and leg .	„	15	„	20	($\frac{1}{6}$)
Inner surface of thigh and leg .	„	25	„	30	($\frac{1}{6}$)
Outer surface of the arm .	„	8	„	33	($\frac{1}{6}$)
Inner surface of the arm .	„	9	„	36	($\frac{1}{6}$)

Manner in which they are disposed in the outline (figure 2, Plate XIX.):—

	Points.				Curves.
Front of neck . . .	from	1	to	2	($\frac{1}{6}$)
„ „ trunk . . .	„	2	„	10	($\frac{1}{4}$)
Back of neck . . .	„	16	„	18	($\frac{1}{6}$)
„ „ trunk . . .	„	18	„	23	($\frac{1}{4}$)
„ „ „	„	23	„	25	($\frac{1}{3}$)
Front of thigh and leg . .	„	11	„	13	($\frac{1}{4}$)
„ „ „ . .	„	13	„	15	($\frac{1}{6}$)
Back of thigh and leg . .	„	25	„	32	($\frac{1}{6}$)
Front of the arm . . .	„	33	„	37	($\frac{1}{6}$)
Back of the arm . . .	„	38	„	40	($\frac{1}{6}$)
Foot	„	0	„	0	($\frac{1}{6}$)

In order to exemplify more clearly the manner in which these various curves appear in the outline of the figure, I give in Plate XX. the whole curvilinear figures, complete, to which these portions belong that form the outline of the sides of the head, neck, and trunk, and of the outer surface of the thighs and legs. Plate XX.

The various angles which the axes of these ellipses form with the vertical, will be found amongst other details in the works I have just referred to.

No. V.

At page 85 I have remarked upon the variety that may be introduced into any particular form of vase; and, in order to give the reader an idea of the ease with which this may be done without violating the harmonic law, I shall here give three examples :—

Plate XXI. The first of these (Plate XXI.) differs from the Portland vase, in the concave curve of the neck flowing more gradually into the convex curve of the body.

Plate XXII. The second (Plate XXII.) differs from the same vase in the same change of contour, as also in being of a smaller diameter at the top and at the bottom.

Plate XXIII. The third (Plate XXIII.) is the most simple arrangement of the elliptic curve by which this kind of form may be produced; and it differs from the Portland vase in the relative proportions of height and diameter, and in having a fuller curve of contour.

The following comparison of the angles employed in these examples, with the angles employed in the original, will shew that the changes of contour in these forms, arise more from the mode in which the angles are arranged than in a change of the angles themselves :—

	Line *A C*	Line *B C*	Line *o p*	Line *v u*	Line *m n*	Line *i k*	ellipse	rectangle
Plate VIII.	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{3}\right)$	(H)	$\left(\frac{3}{10}\right)$	$\left(\frac{1}{3}\right)$	$\left(\frac{1}{5}\right)$	$\left(\frac{1}{4}\right)$	$\left(\frac{2}{5}\right)$
Plate XXI. ...	$\left(\frac{1}{2}\right)$ ...	$\left(\frac{1}{3}\right)$ ...	$\left(\frac{2}{9}\right)$ ...	$\left(\frac{1}{4}\right)$ ...	$\left(\frac{2}{9}\right)$ ...	$\left(\frac{1}{5}\right)$ ...	$\left(\frac{1}{4}\right)$ ...	$\left(\frac{2}{5}\right)$
Plate XXII. ...	$\left(\frac{1}{2}\right)$ ...	$\left(\frac{1}{3}\right)$ ...	$\left(\frac{1}{8}\right)$ ...	$\left(\frac{4}{9}\right)$ ...	$\left(\frac{1}{3}\right)$ ...	$\left(\frac{1}{5}\right)$ ...	$\left(\frac{1}{4}\right)$ ...	$\left(\frac{2}{5}\right)$
Plate XXIII. ...	$\left(\frac{1}{2}\right)$ ...	$\left(\frac{1}{4}\right)$ ...	(H) ...	(—) ...	$\left(\frac{1}{5}\right)$ ...	$\left(\frac{1}{5}\right)$	ellipses $\left\{\left(\frac{1}{3}\right) \left(\frac{1}{4}\right)\right\}$ ...	$\left(\frac{1}{3}\right)$

The harmonic elements of each are therefore simply the following parts of the right angle :—

	Tonic.	Dominant.	Mediant.	Submediant.
Plate VIII.	$\left(\frac{1}{2}\right)$ $\left(\frac{1}{4}\right)$	$\left(\frac{1}{3}\right)$	$\left(\frac{1}{5}\right)$	$\left(\frac{3}{10}\right)$

	Tonic.	Dominant.	Mediant.	Supertonic.
Plate XXI.	$\left(\frac{1}{2}\right)$ $\left(\frac{1}{4}\right)$	$\left(\frac{1}{3}\right)$	$\left(\frac{1}{5}\right)$	$\left(\frac{2}{9}\right)$

	Tonic.	Dominant.	Mediant.	Supertonic.
Plate XXII.	($\frac{1}{2}$)	($\frac{1}{3}$)	($\frac{1}{5}$)	($\frac{4}{9}$)
	($\frac{1}{4}$)			
	($\frac{1}{8}$)			

	Tonic.	Dominant.	Mediant.
Plate XXIII.	($\frac{1}{2}$)	($\frac{1}{3}$)	($\frac{1}{5}$)
	($\frac{1}{4}$)		

No. VI.

So far as I know, there has been only one attempt in modern times, besides my own, to establish a universal system of proportion, based on a law of nature, and applicable to art. This attempt consists of a work of 457 pages, with 166 engraved illustrations, by Dr Zeising, a professor in Leipzic, where it was published in 1854.

One of the most learned and talented professors in our Edinburgh University has reviewed that work as follows :—

"It has been rather cleverly said that the intellectual distinction between an Englishman and a Scotchman is this—'Give an Englishman two facts, and he looks out for a third; give a Scotchman two facts, and he looks out for a theory.' Neither of these tests distinguishes the German; he is as likely to seek for a third fact as for a theory, and as likely to build a theory on two facts as to look abroad for further information. But once let him have a theory in his mind, and he will ransack heaven and earth until he almost buries it under the weight of accumulated facts. This remark applies with more than common force to a treatise published last year by Dr Zeising, a professor in Leipsic, 'On a law of proportion which rules all nature.' The ingenious author, after proving from the writings of ancient and modern philosophers that there always existed the belief (whence derived it is difficult to say), that some law does bind into one formula all the visible works of God, proceeds to criticise the opinions of individual writers respecting that connecting law. It is not our purpose to follow him through his lengthy examination. Suffice it to say that he believes he has found the lost treasure in the *Timæus* of Plato, c. 31. The passage is confessedly an obscure one, and will not bear a literal translation. The interpretation which Dr Zeising puts on it is certainly a little strained, but we are disposed to admit that he does it with considerable reason. Agreeably to him, the passage runs thus :— 'That bond is the most beautiful which binds the things as much as possible into one; and proportion effects this most perfectly when three things are so united that the greater bears to the middle the same ratio that the middle bears to the less.'

"We must do Dr Zeising the justice to say that he has not made more than a legitimate use of the materials which were presented to him in the writings of the ancients, in his endeavour to establish the fact of the existence of this law amongst them. The canon of Polycletes, the tradition of Varro mentioned by Pliny relative to that canon, the writings of Galen and others, are all brought to bear on the same point with more or less force. The sum of this portion of the argument is fairly this,—that the ancient sculptors had *some* law of proportion—some authorised examplar to which they referred as their work proceeded. That it was the law here attributed to Plato is by no means made out; but, considering the incidental manner in which that law is referred to, and the obscurity of the passages as they exist, it is, perhaps, too much to expect more than this broad feature of coincidence—the fact that some law was known and appealed to. Dr Zeising now proceeds to examine modern theories, and it is fair to state that he appears generally to take a very just view of them.

"Let us now turn to Dr Zeising's own theory. It is this—that in every beautiful form lines are divided in extreme and mean ratio; or, that any line considered as a whole, bears to its larger part the same proportion that the larger bears to the smaller—thus, a line of 5 inches will be divided into parts which are very nearly 2 and 3 inches respectively (1·9 and 3·1 inches). This is a well-known division of a line, and has been called the GOLDEN rule, but when or why, it is not easy to ascertain. With this rule in his hand, Dr Zeising proceeds to examine all nature and art; nay, he even ventures beyond the threshold of nature to scan Deity. We will not follow him so far. Let us turn over the pages of his carefully illustrated work, and see how he applies his line. We meet first with the Apollo Belvidere—the golden line divides him happily. We cannot say the same of the division of a handsome face which occurs a little further on. Our preconceived notions have made the face terminate with the chin, and not with the centre of the throat. It is evident that, with such a rule as this, a little latitude as to the extreme point of the object to be measured, relieves its inventor from a world of perplexities. This remark is equally applicable to the *arm* which follows, to which the rule appears to apply admirably, yet we have tried it on sundry plates of arms, both fleshy and bony, without a shadow of success. Whether the rule was made for the arm or the arm for the rule, we do not pretend to decide. But let us pass hastily on to page 284, where the Venus de Medicis and Raphael's Eve are presented to us. They bear the application of the line right well. It might, perhaps, be objected that it is remarkable that the same rule applies so exactly to the existing position of the figures, such as the Apollo and the Venus, the one of which is upright, and the other crouching. But let that pass. We find Dr Zeising next endeavouring to square his theory with the distances of the planets, with wofully scanty success. Descending from his lofty position, he spans the earth from corner to corner, at which occupation we will leave him for a moment, whilst we offer a suggestion which is equally applicable to poets, painters, novelists, and theorisers. Never err in excess—defect is the safe side—it is seldom a fault, often a real merit. Leave something for the student of your works to do—don't chew the cud for him. Be assured he will not omit to pay you for every little thing which you have enabled him to discover. Poor Professor Zeising! he is far too German to leave any field of discovery open for his readers. But let us return to him; we left him on his back, lost for a time in a hopeless attempt to double Cape Horn. We will be kind to him, as the child is to his man in the Noah's ark, and set him on his legs amongst his toys again. He is now in the vegetable kingdom, amidst oak leaves and sections of the stems of divers plants. He is in his element once more, and it were ungenerous not to admit the merit of his endeavours, and the success which now and then attends it. We will pass over his horses and their riders, together with that portly personage, the Durham ox, for we have caught a glimpse of a form familiar to our eyes, the ever-to-be-admired Parthenon. This is the true test of a theory. Unlike the Durham ox just passed before us, the Parthenon will stand still to be measured. It has so stood for twenty centuries, and every one that has scanned its proportions has pronounced them exquisite. Beauty is not an adaptation to the acquired taste of a single nation, or the conventionality of a single generation. It emanates from a deep-rooted principle in nature, and appeals to the verdict of our whole humanity. We don't find fault with the Durham ox—his proportions are probably good, though they be the result of breeding and cross-breeding; still we are not sure whether, in the march of agriculture, our grandchildren may not think him a very wretched beast. But there is no mistake about the Parthenon; as a type of proportion it stands, has stood, and shall stand. Well, then, let us see how Dr Zeising succeeds with his rule here. Alas! not a single point comes right. The Parthenon is condemned, or its condemnation condemns the theory. Choose your part. We choose the latter alternative; and now, our choice being made, we need proceed no further. But a question or two have presented themselves as we went along, which demand an answer. It may be asked—How do you account for the esteem in which this law of the section in extreme and mean ratio was held? We reply—That it was esteemed just in the same way that a tree is esteemed for its fruit. To divide a right angle into two or three, four or six, equal parts was easy enough. But to divide it into five or ten such parts was a real difficulty. And how was the difficuty got over? It was effected by means of this golden rule. This is its great, its ruling application; and if we adopt the notion that the ancients were possessed with the idea of the existence of angular symmetry, we shall have no difficulty in accounting for their appreciation of this problem. Nay, we may even go further, and admit, with Dr Zeising, the interpretation of the passage of Plato,—only with this limitation, that Plato, as a geometer, was carried away by the geometry of æsthetics from the thing itself. It may be asked again—Is it not probable that *some* proportionality does exist amongst the parts of natural objects? We reply—That, *à priori*, we expect some such system to exist, but that it is inconsistent with the scheme of *least effort*, which pervades and characterises all natural succession in space or in time, that that system should be a complicated one. Whatever it is, its essence must be simplicity. And no system of simple linear proportion is found in nature; quite the contrary. We are, therefore, driven to another hypothesis, viz.—that the simplicity is one of angles, not of lines; that the eye estimates by search round a point, not by ascending and descending, going to the right and to the left,—a theory which we conceive all nature conspires to prove. Beauty was not created for the eye of man, but the eye of man and his mental eye were created for the appreciation of beauty. Examine the forms of animals and plants so minute that nothing short of the most recent improvements in the microscope can succeed in detecting their symmetry; or examine the forms of those little silicious creations which grew thousands of years before Man was placed on the earth, and, with forms of marvellous and varied beauty, they all point to its source in angular symmetry. This is the keystone of formal beauty, alike in the minutest animalcule, and in the noblest of God's works, his own image—Man."

THE END.

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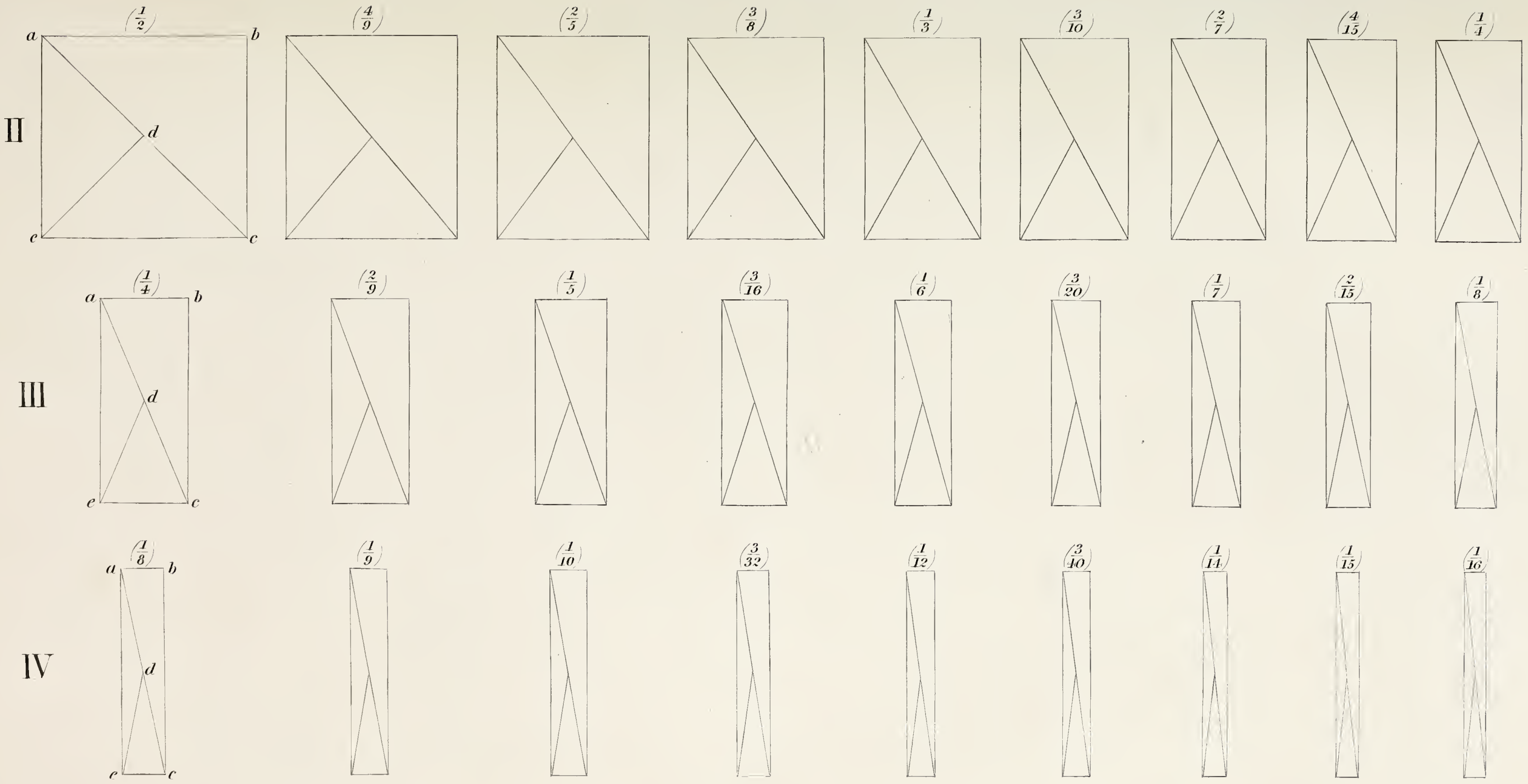

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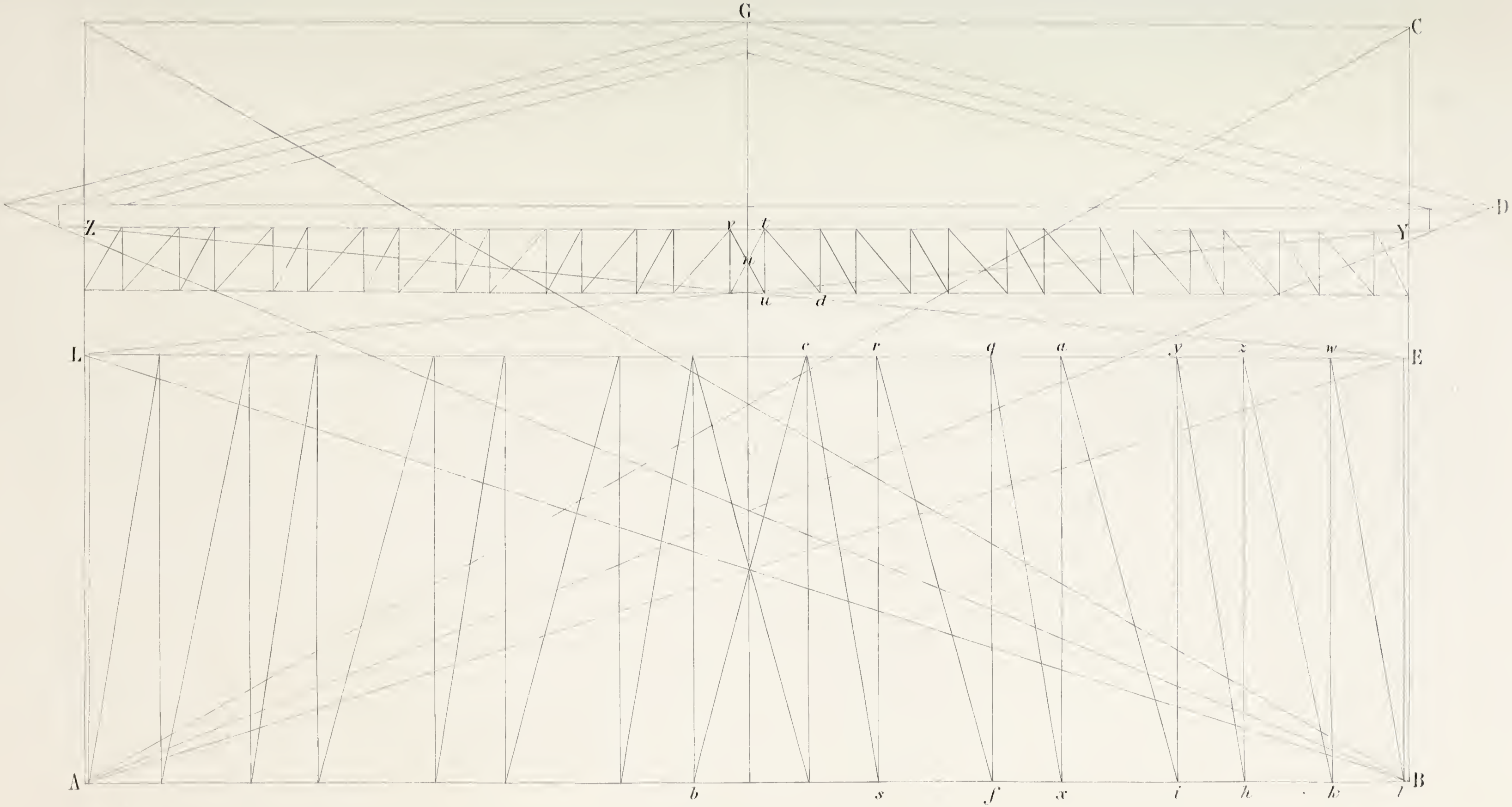

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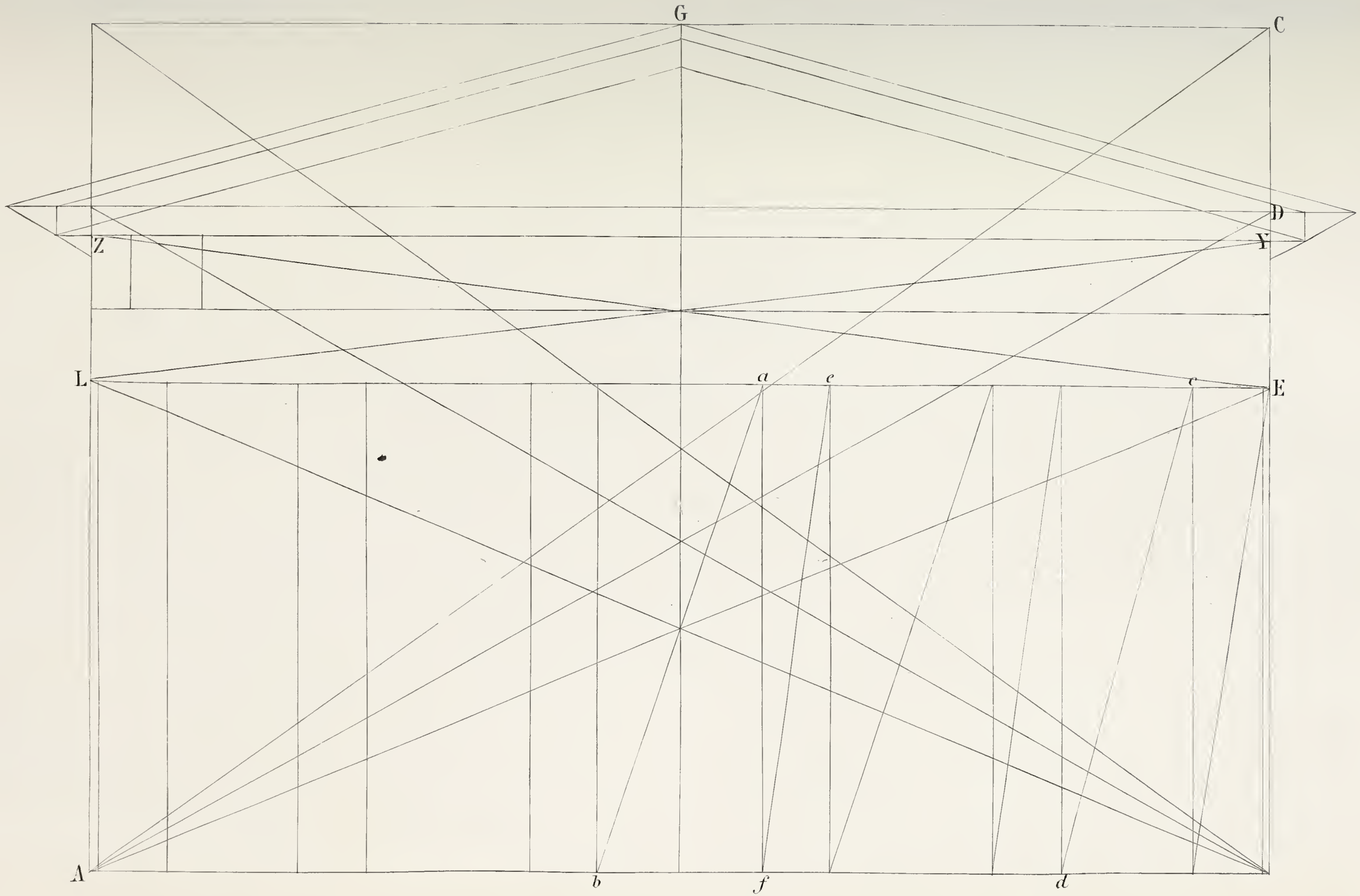

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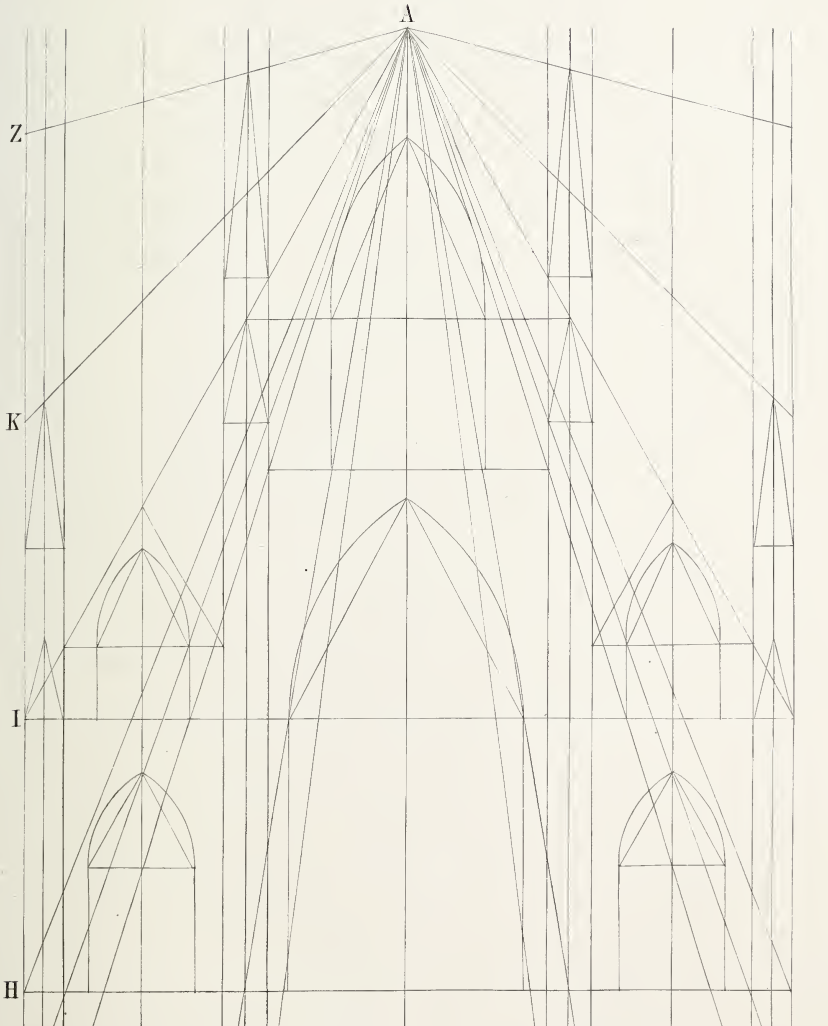

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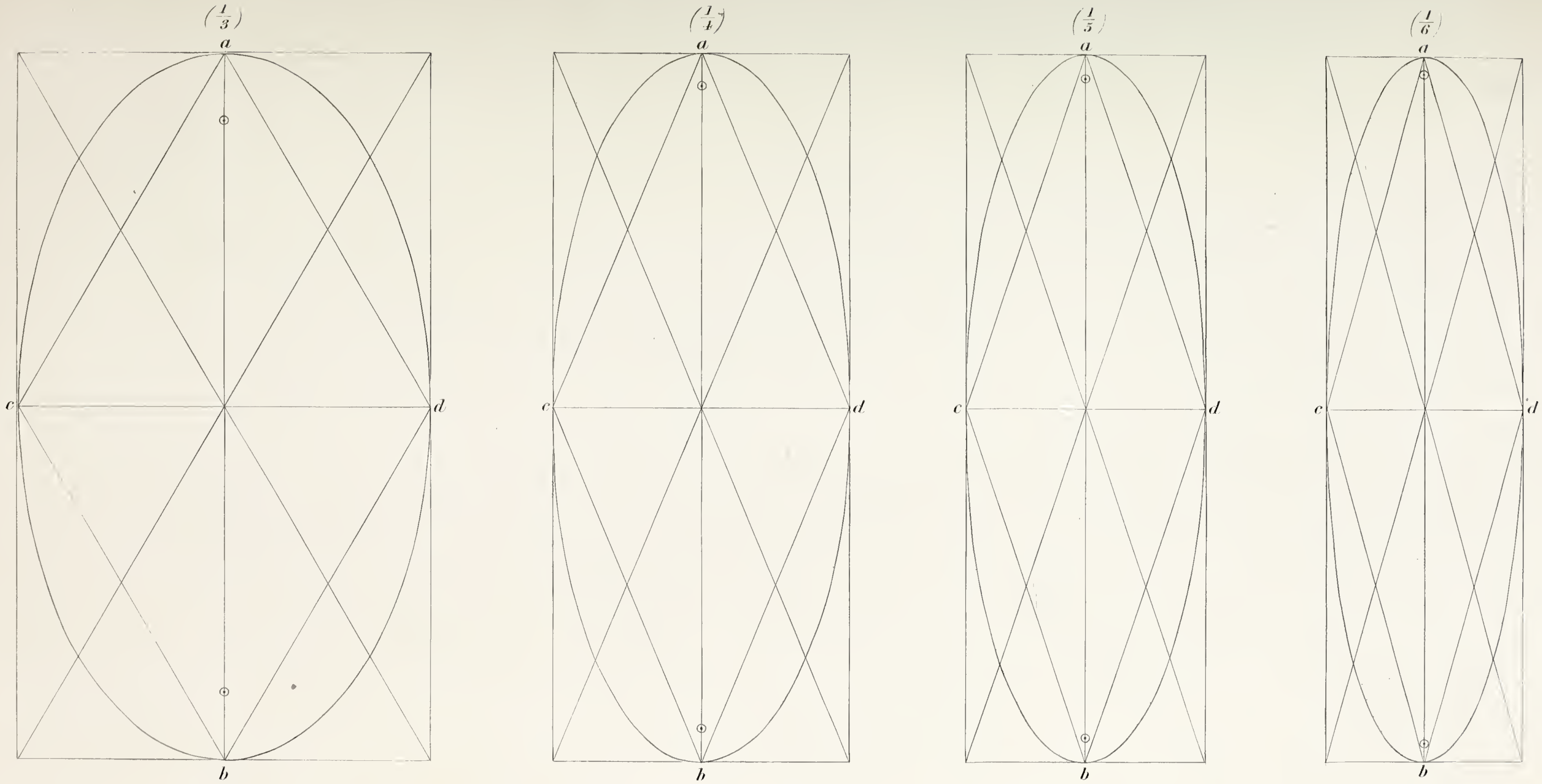

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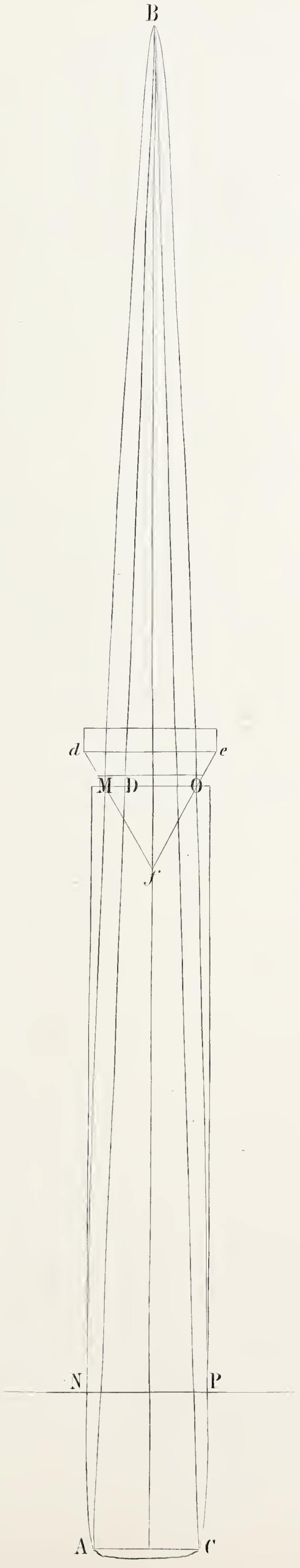

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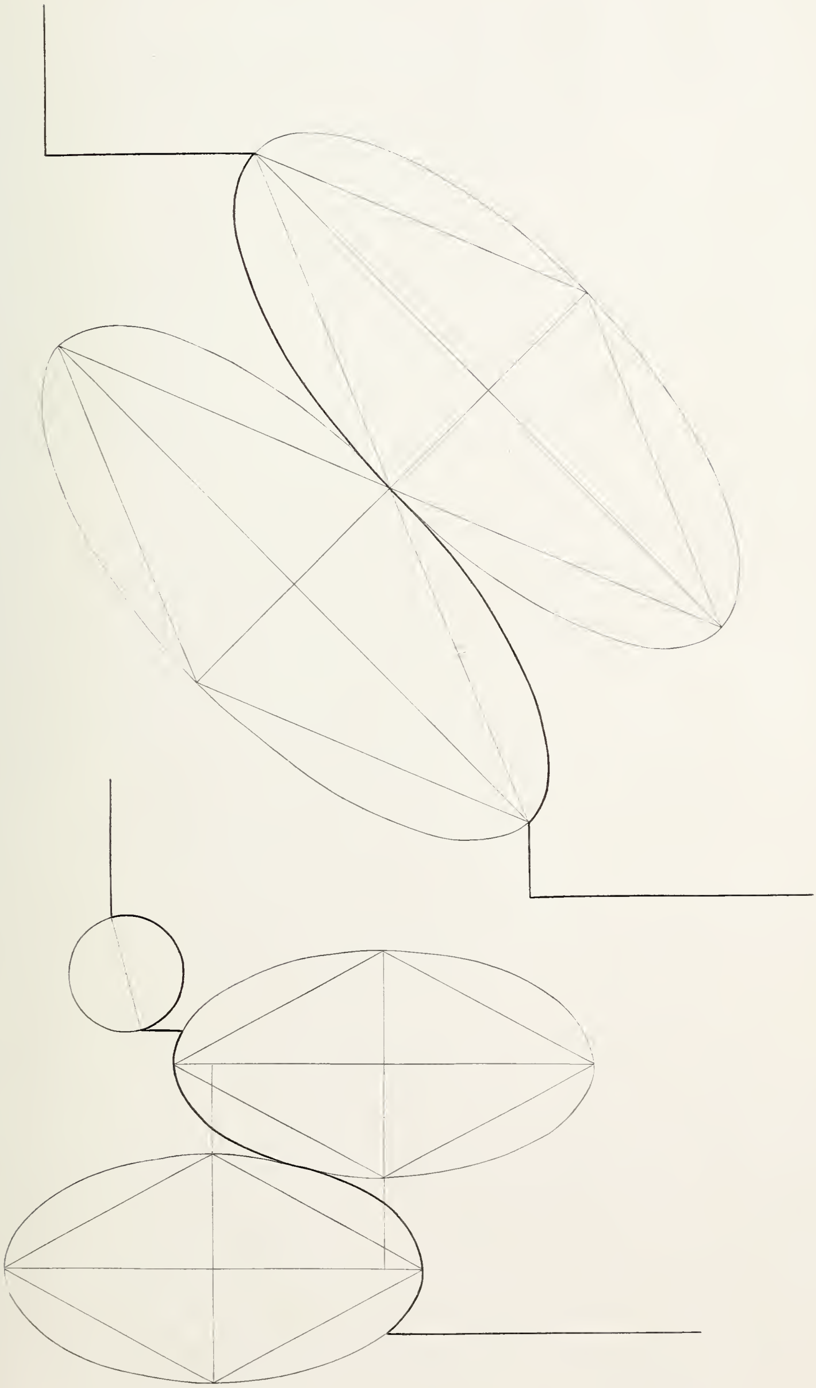

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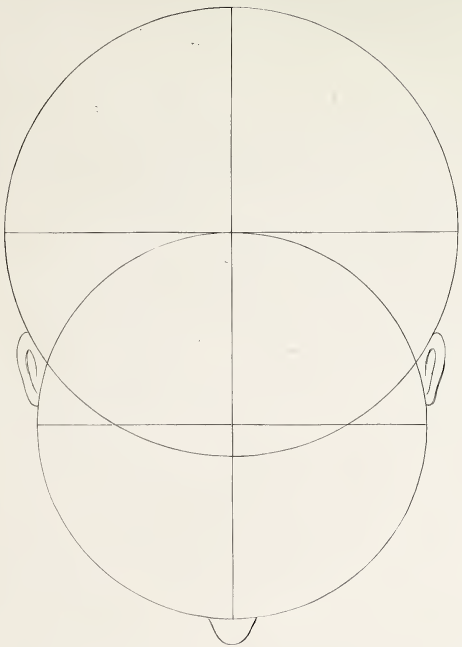

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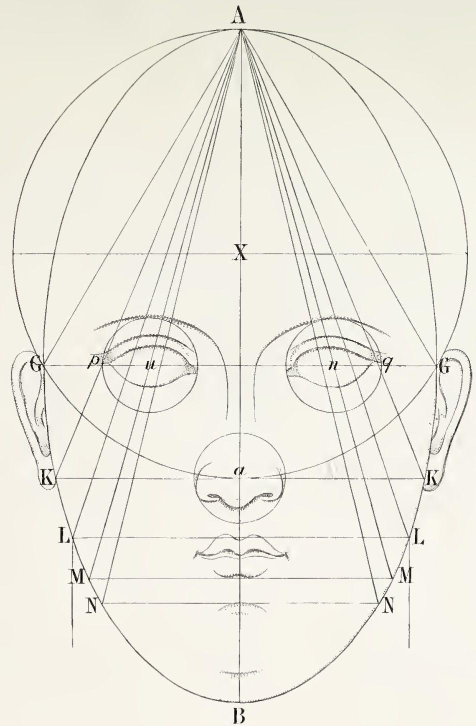

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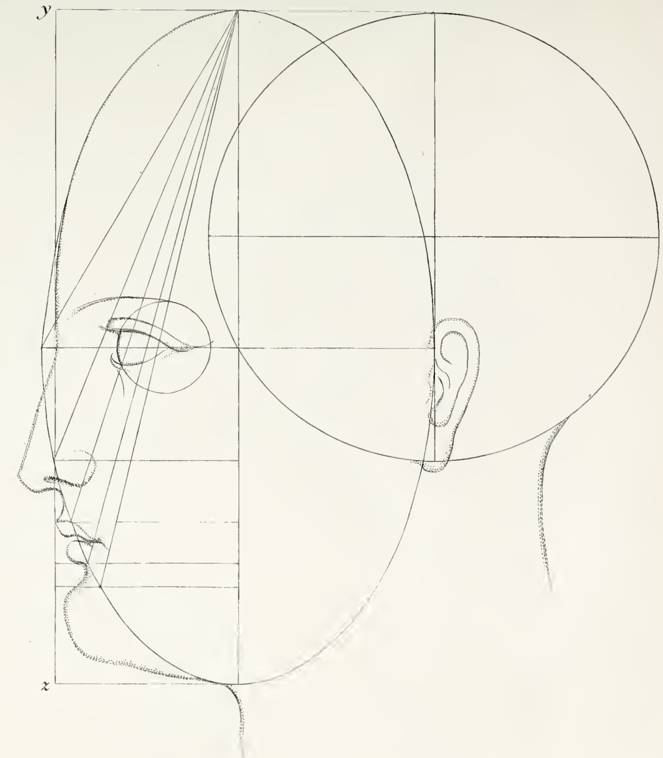

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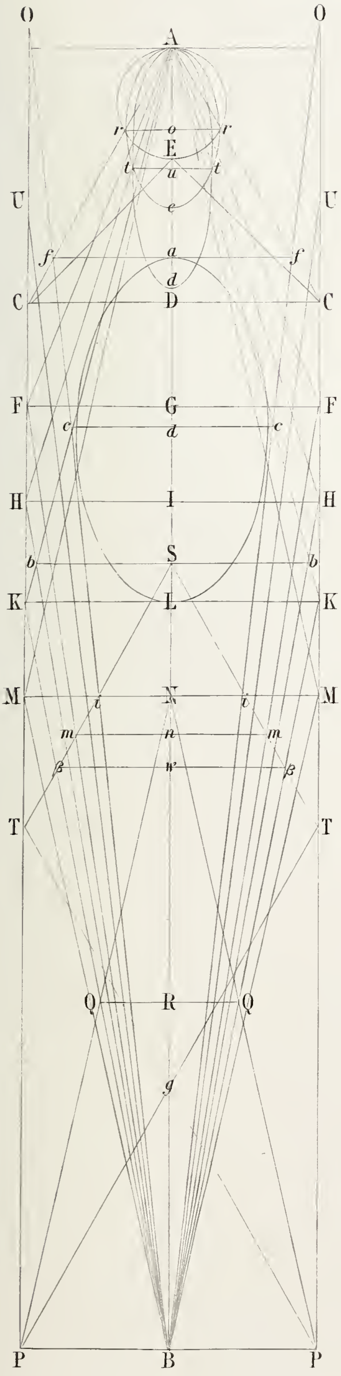

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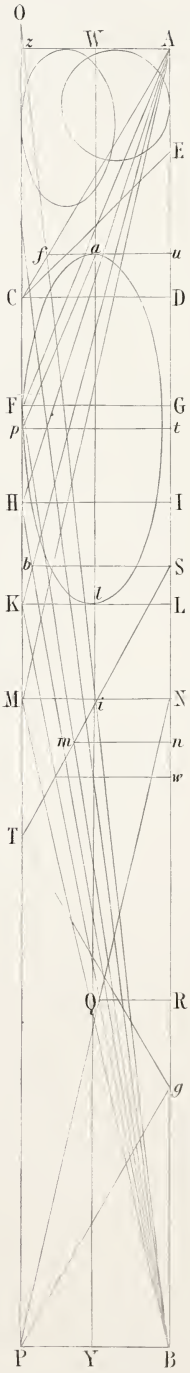

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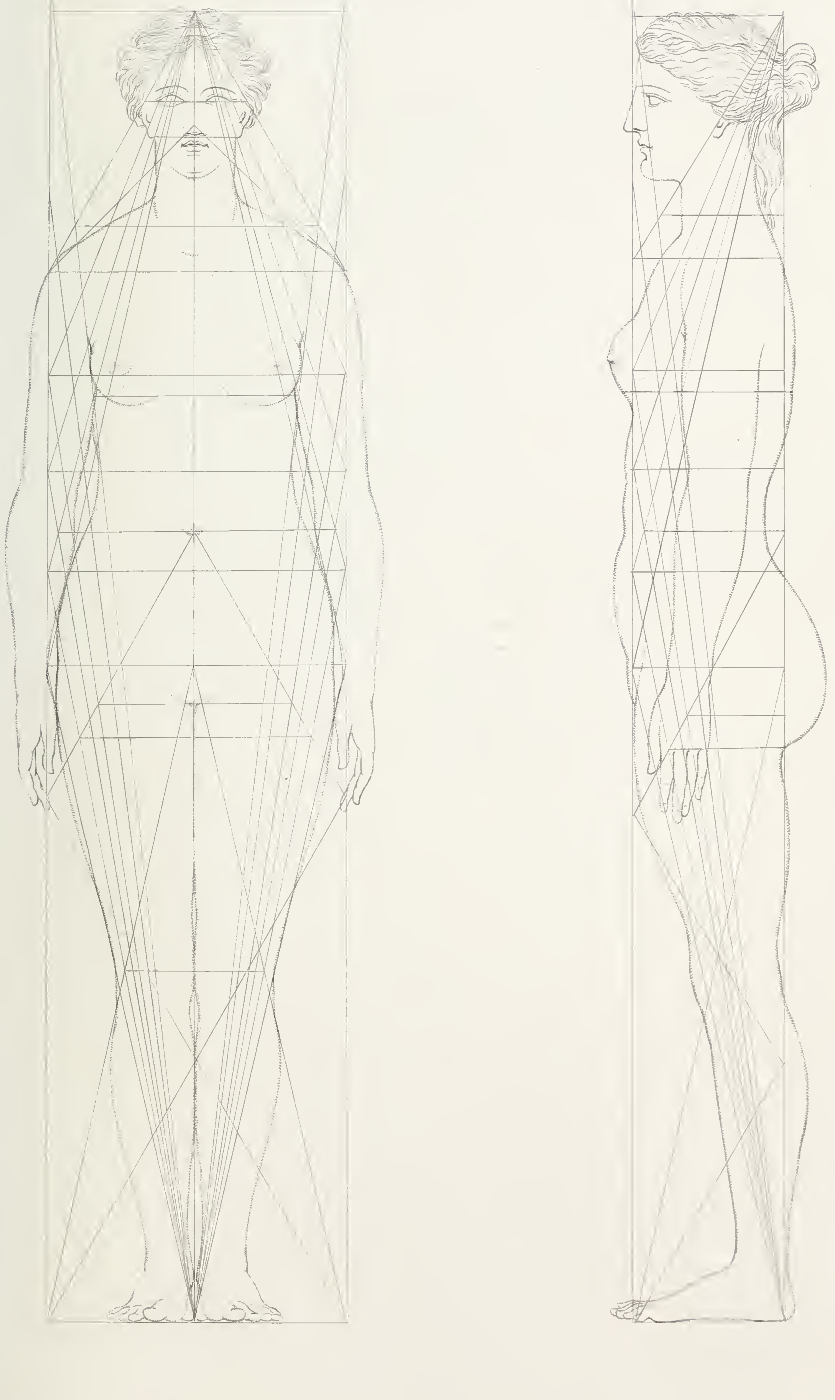

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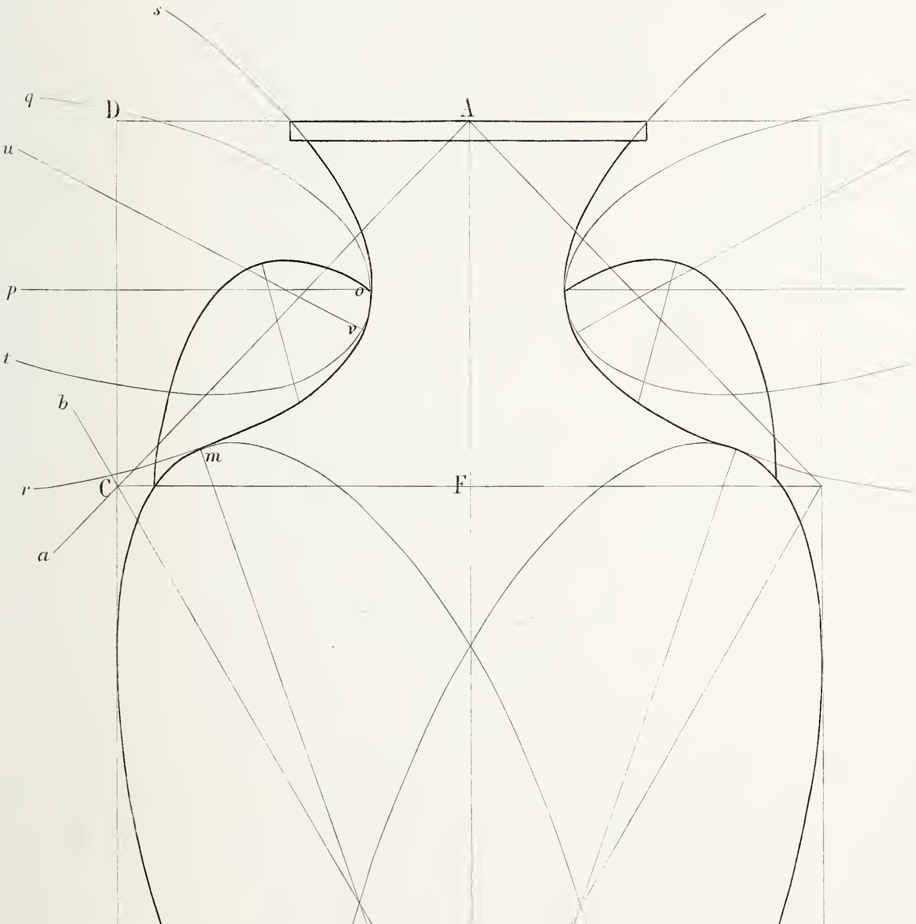

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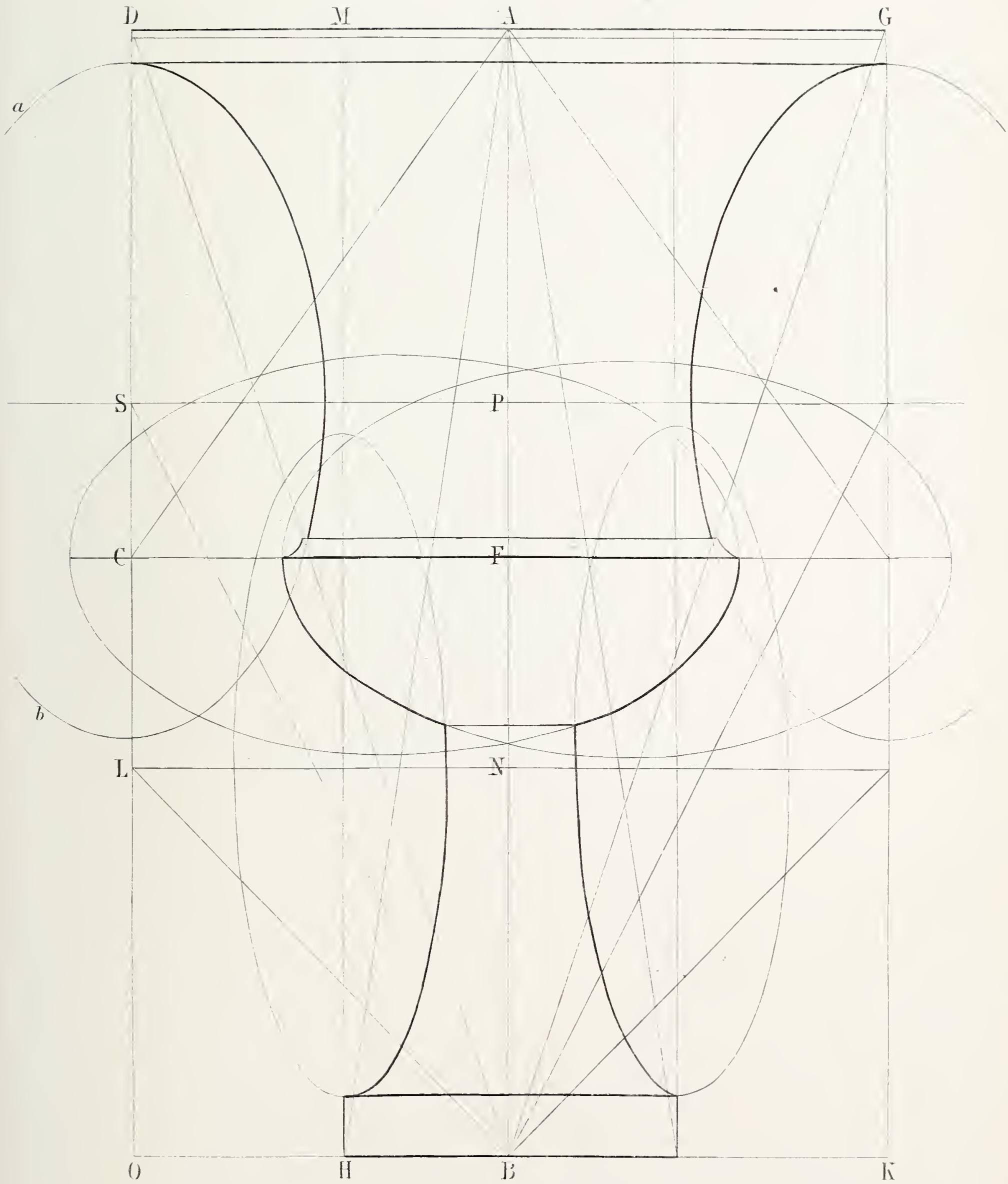

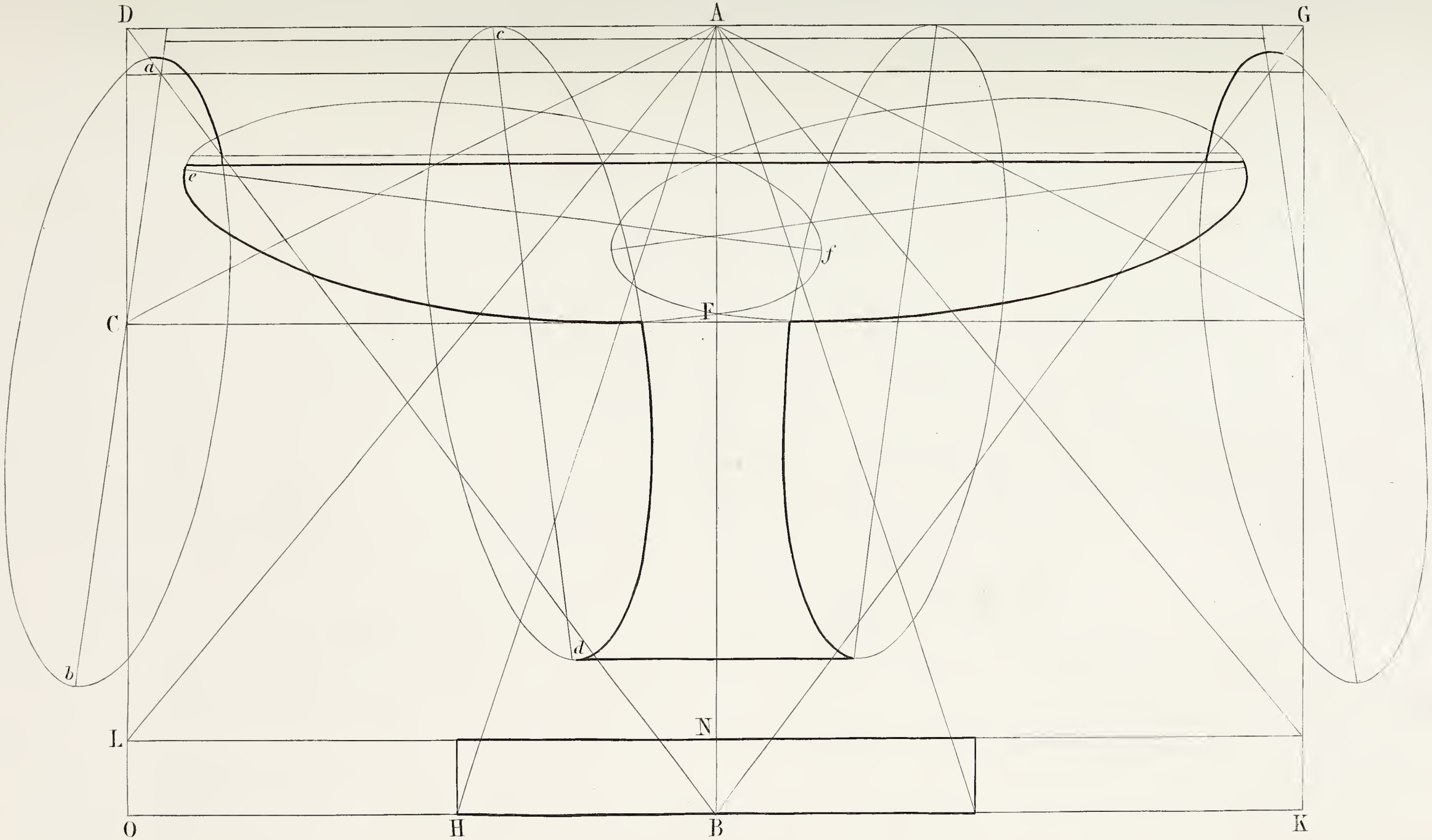

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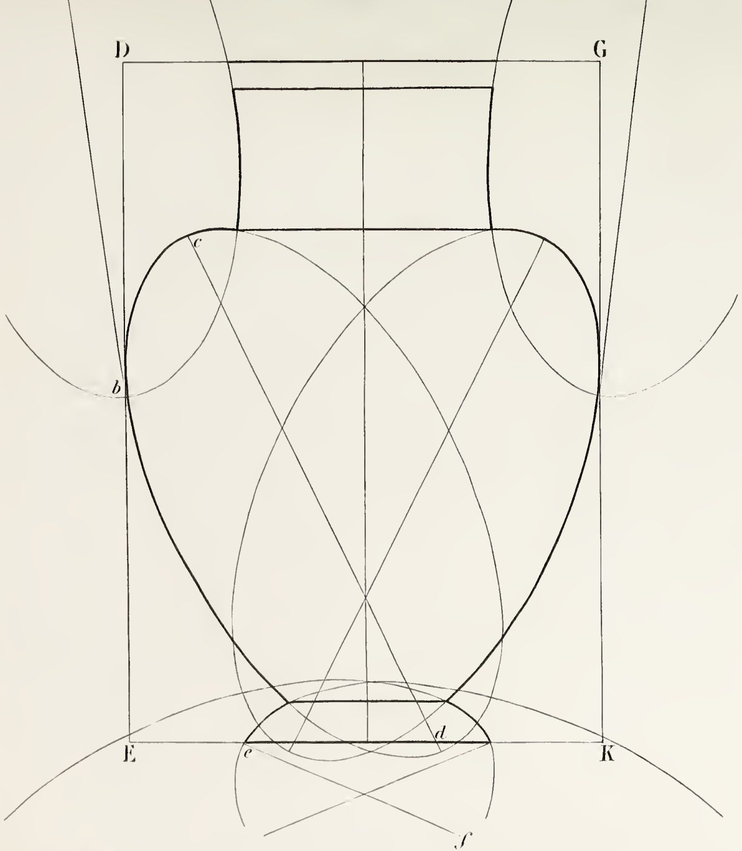

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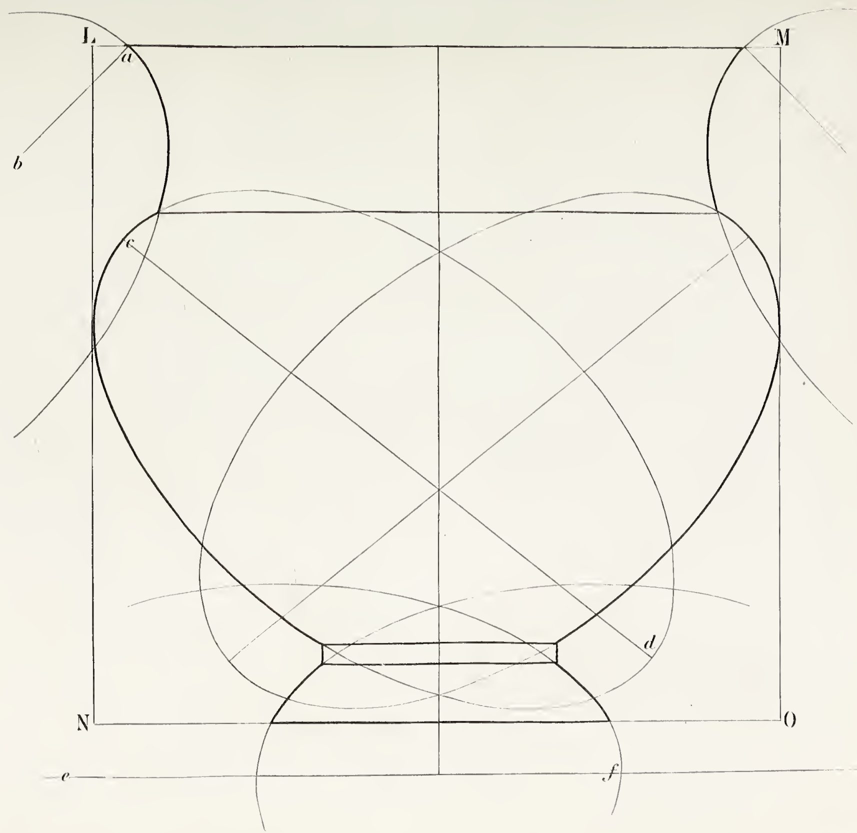

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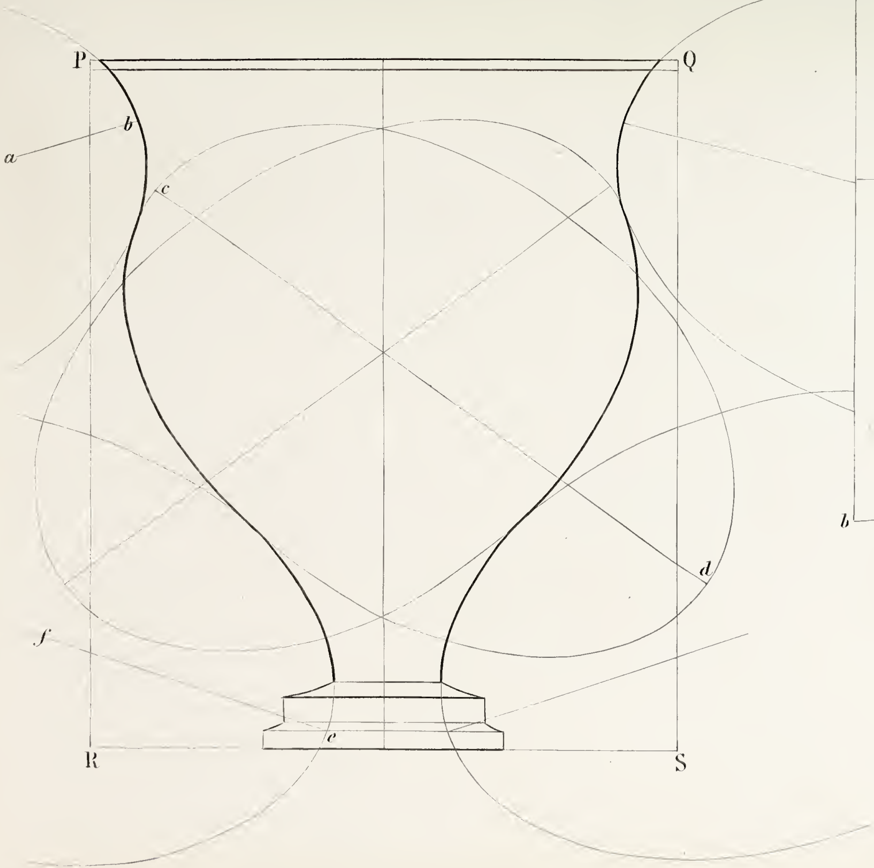

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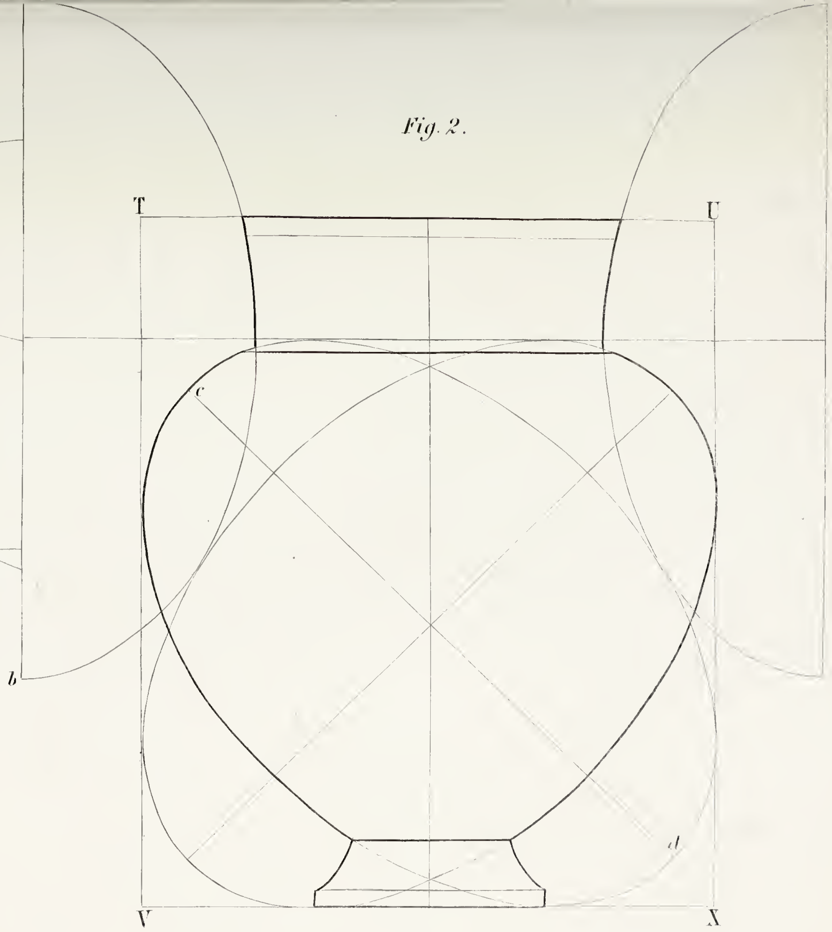

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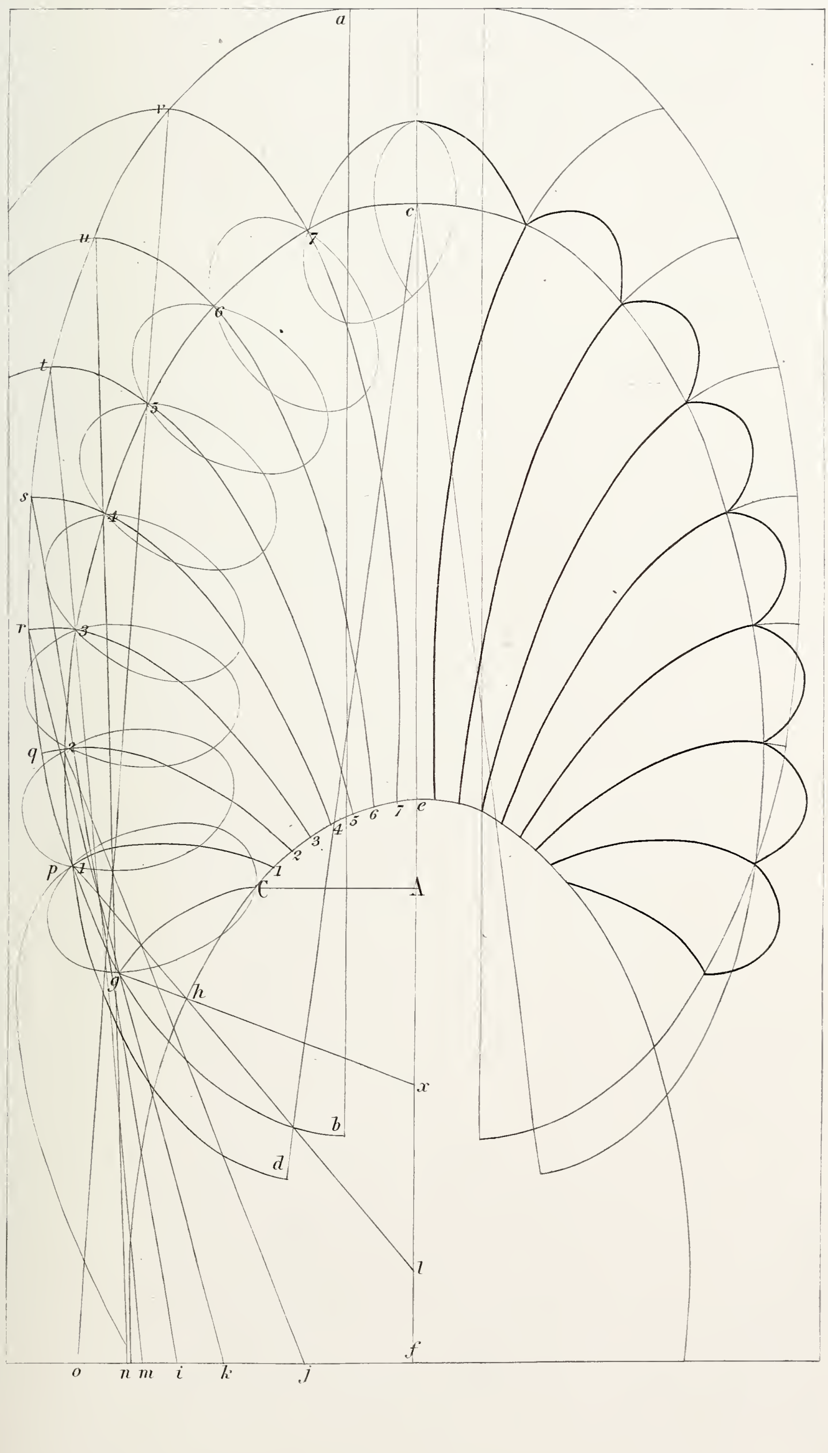

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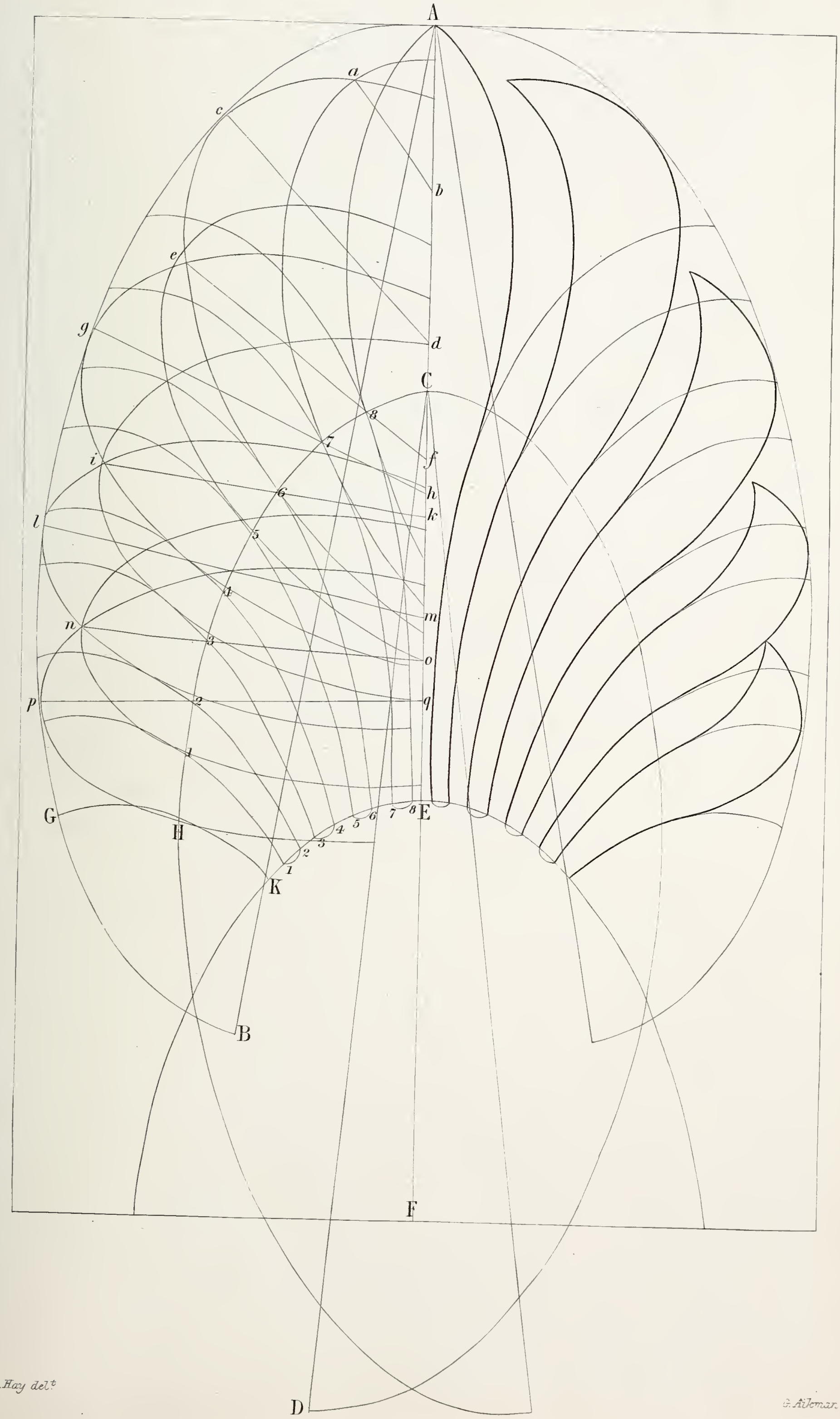

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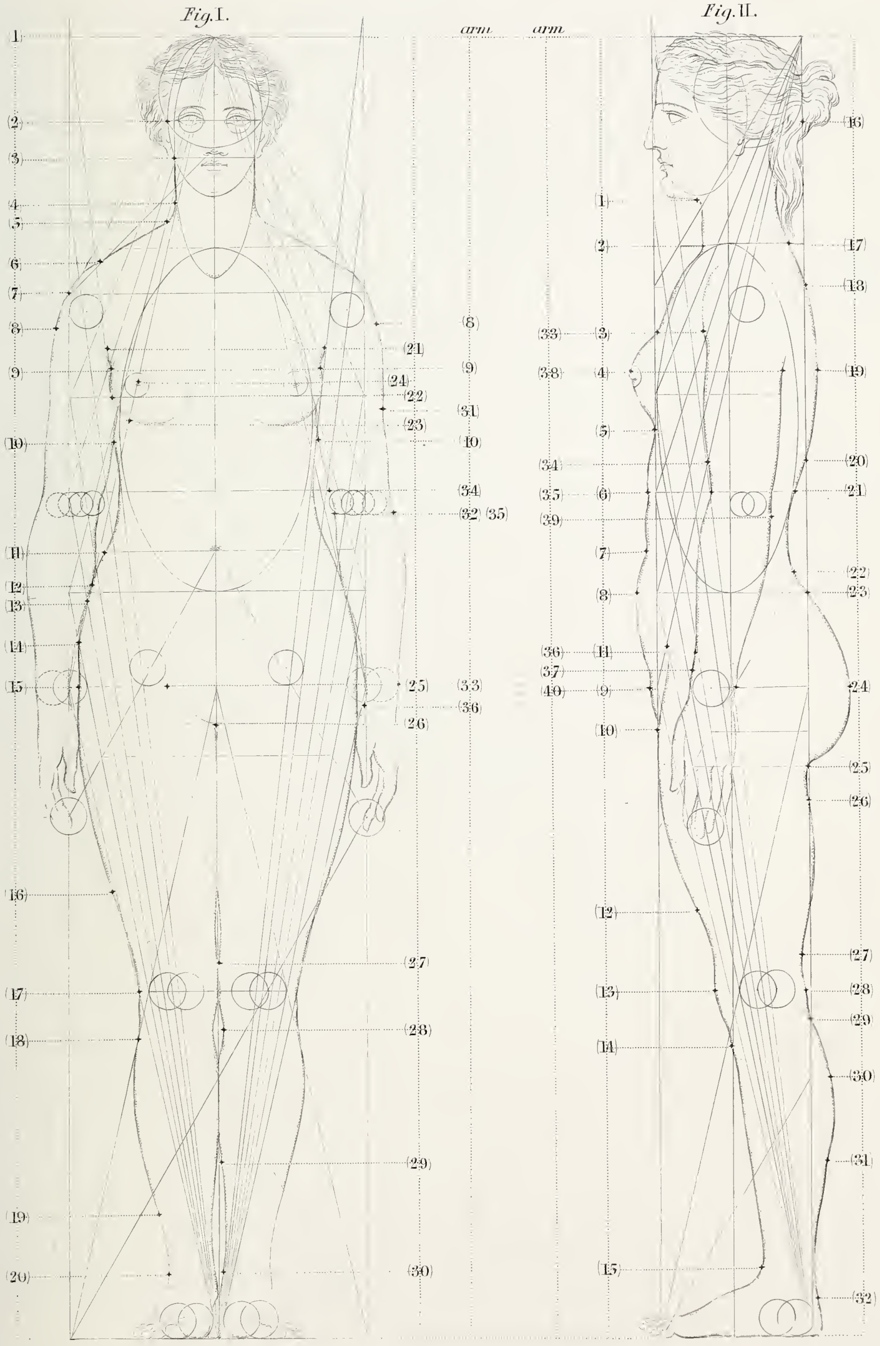

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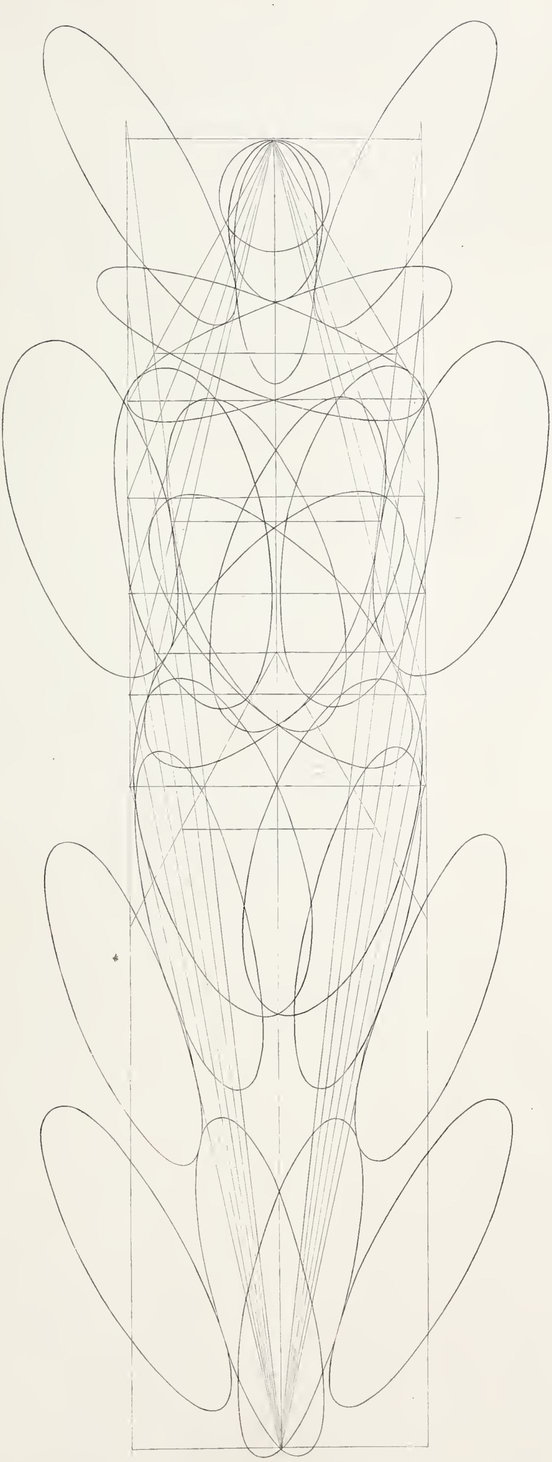

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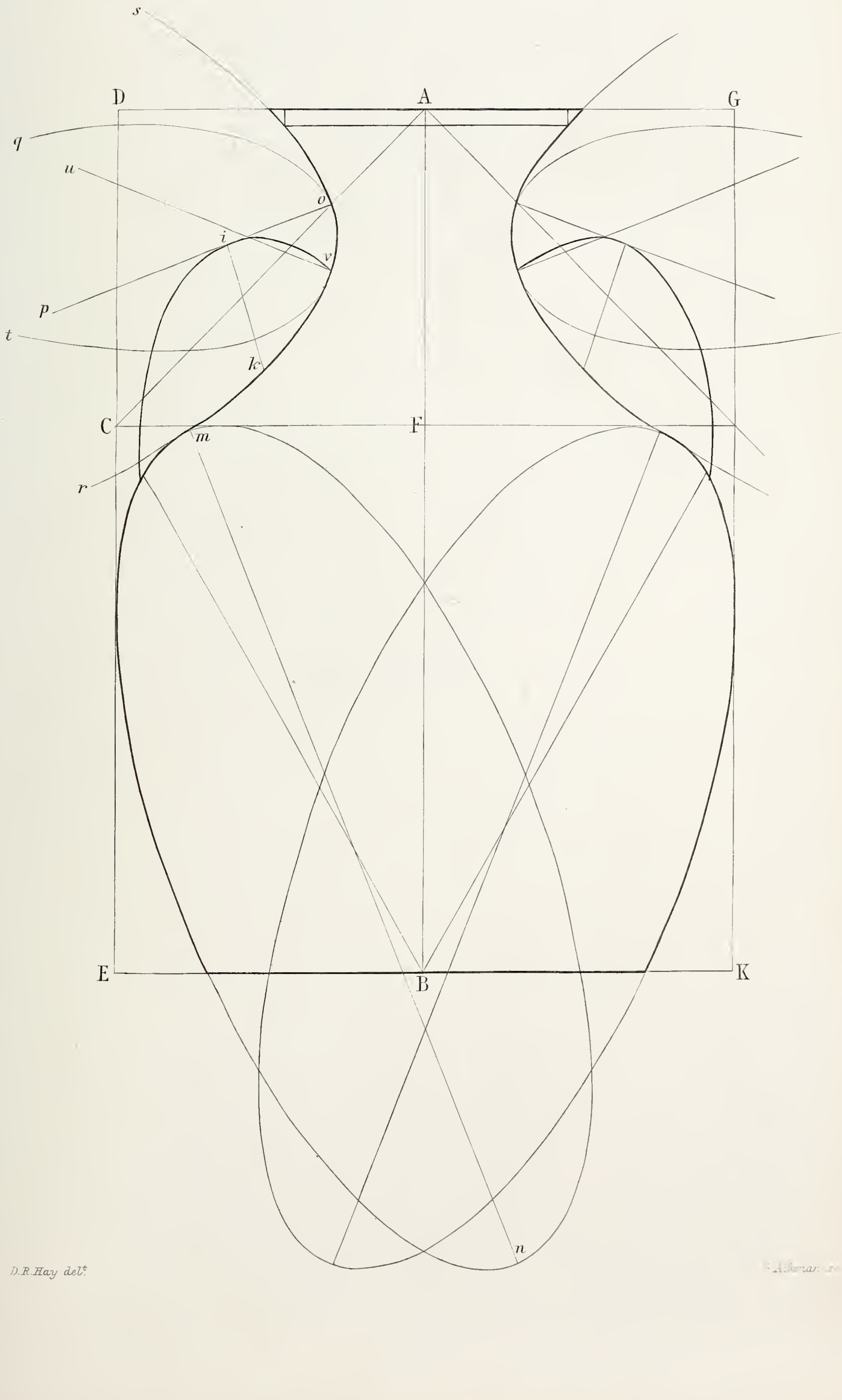

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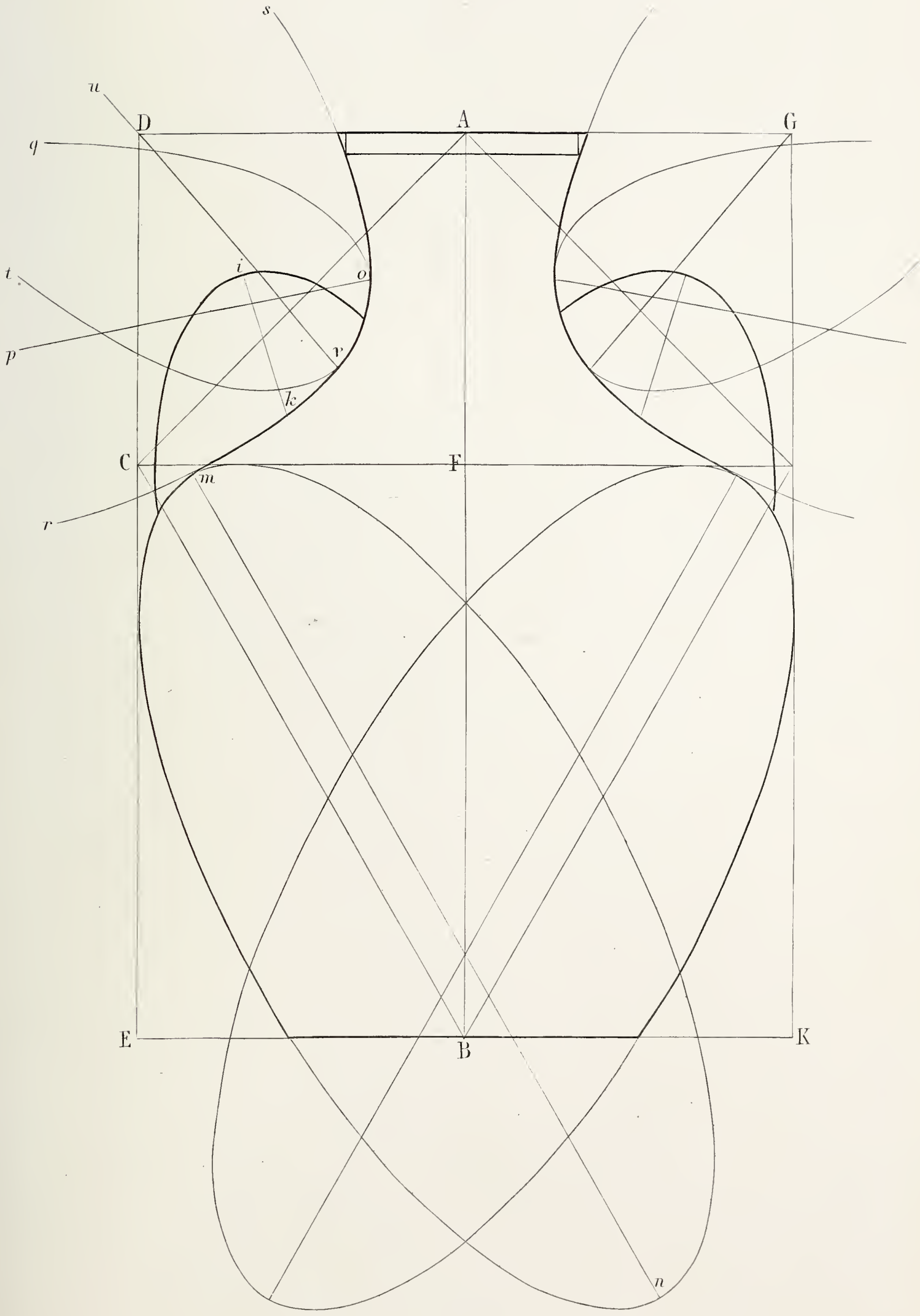

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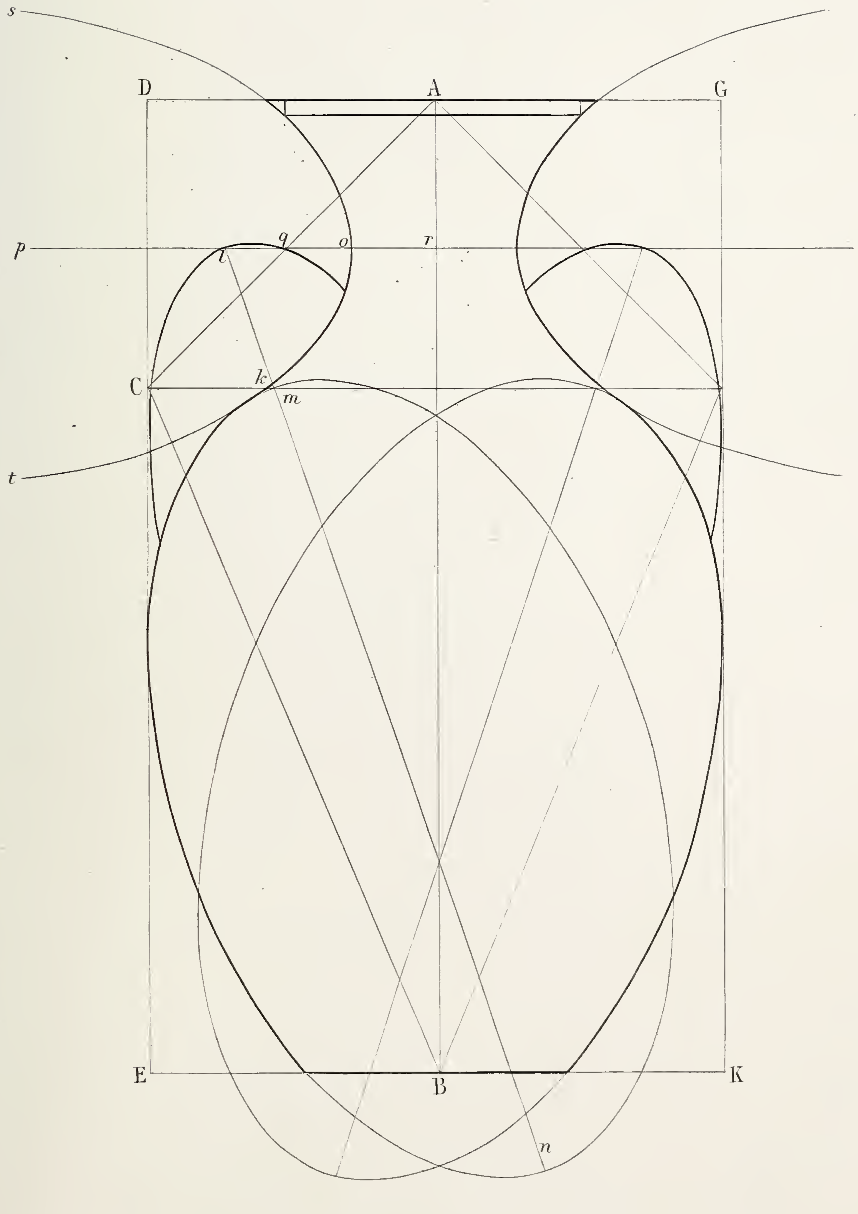

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Again and again the attempt has been made to detect harmonic ratios in the measurement of Athenian architecture, but ever without reward. Mr Hay has, however, made the discovery, and to an extent of which no one had previously dreamt.

IV.

In 8vo, 100 Plates, price 6s.,

FIRST PRINCIPLES OF SYMMETRICAL BEAUTY.

From the Spectator.

This is a grammar of pure form, in which the elements of symmetrical, as distinguished from picturesque beauty, are demonstrated, by reducing the outlines or planes of curvilinear and rectilinear forms to their origin in the principles of geometrical proportion. In thus analysing symmetry of outline in natural and artificial objects, Mr Hay determines the fixed principles of beauty in positive shape, and shews how beautiful forms may be reproduced and infinitely varied with mathematical precision. Hitherto the originating and copying of beautiful forms have been alike empirical; the production of a new design for a vase or a jug has been a matter of chance between the eye and the hand; and the copying of a Greek moulding or ornament, a merely mechanical process. By a series of problems, Mr Hay places both the invention and imitation of beautiful forms on a sure basis of science, giving to the fancy of original minds a clue to the evolving of new and elegant shapes, in which the infinite resources of nature are made subservient to the uses of art.

The volume is illustrated by one hundred diagrams beautifully executed, that serve to explain the text, and suggest new ideas of beauty of contour in common objects. To designers of pottery, hardware, and architectural ornaments, this work is particularly valuable; but artists of every kind, and workmen of intelligence, will find it of great utility.

From the Athenæum.

The volume before us is the seventh of Mr Hay's works. It is the most practical and systematic, and likely to be one of the most useful. It is, in short, a grammar of form, or a spelling-book of beauty. This is beginning at the right end of the matter; and the necessity for this kind of knowledge will inevitably, though gradually, be felt. The work will, therefore, be ultimately appreciated and adopted as an introduction to the study of beautiful forms.

The third part of the work treats of the Greek oval or composite ellipse, as Mr Hay calls it. It is an ellipse of three foci, and gives practical forms for vases and architectural mouldings, which are curious to the architect, and will be very useful both to the potter, the moulder, and the pattern-drawer. A fourth part contains applications of this to practice. Of the details worked out with so much judgment and ingenuity by Mr Hay, we should in vain attempt to communicate just notions without the engravings of which his book is full. We must, therefore, refer to the work itself. The forms there given are full of beauty, and so far tend to prove the system.

V.

In 8vo, 14 Coloured Diagrams, Second Edition, price 15s.,

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From the Spectator.

In this new analysis of the harmonies of colour, Mr Hay has performed the useful service of tracing to the operation of certain fixed principles the sources of beauty in particular combinations of hues and tints; so that artists may, by the aid of this book, produce, with mathematical certainty, the richest effects, hitherto attainable by genius alone. Mr Hay has reduced this branch of art to a perfect system, and proved that an offence against good taste in respect to combinations of colour is, in effect, a violation of natural laws.

VI.

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From the Daily News.

In this work Mr Hay has brought a larger amount of practical knowledge to bear on the subject of colour than any other writer with whom we are acquainted, and in proportion to this practical knowledge is, as might be expected, the excellence of his treatise. There is much in this volume which we would most earnestly recommend to the notice of artists, house decorators, and, indeed, to all whose business or profession requires a knowledge of the management of colour. The work is replete with hints which they might turn to profitable account, and which they will find nowhere else.

From the Athenæum.

We have formerly stated the high opinion we entertain of Mr Hay's previous exertions for the improvement of decorative art in this country. We have already awarded him the merit of invention and creation of the new and the beautiful in form. In his former treatises he furnished a theory of definite proportions for the creation of the beautiful in form. In the present work he proposes to supply a scale of definite proportions for chromatic beauty. For this purpose he sets out very properly with a precise nomenclature of colour. In this he has constructed a vocabulary for the artist—an alphabet for the artizan. He has gone further —he constructs words for three syllables. From this time, it will be possible to write a letter in Edinburgh about a coloured composition, which shall be read off in London, Paris, St Petersburg, or Pekin, and shall so express its nature that it can be reproduced in perfect identity. This Mr Hay has done, or at least so nearly, as to deserve our thanks on behalf of art, and artists of all grades, even to the decorative artizan—not one of whom, be he house-painter, china pattern-drawer, or calico printer, should be without the simple manual of "words for colours."

VII.

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From the Edinburgh Review.

In so far as we know, Mr Hay is the first and only modern artist who has entered upon the study of these subjects without the trammels of prejudice and authority. Setting aside the ordinances of fashion, as well as the dicta of speculation, he has sought the foundation of his profession in the properties of light, and in the laws of visual sensation, by which these properties are recognised and modified. The truths to which he has appealed are fundamental and irrefragable.

From the Athenæum.

We have regarded, and do still regard, the production of Mr Hay's works as a remarkable psychological phenomenon—one which is instructive both for the philosopher and the critic to study with care and interest, not unmingled with respect. We see how his mind has been gradually guided by Nature herself out of one track, and into another, and ever and anon leading him to some vein of the beautiful and true, hitherto unworked.

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Mr Hay has studied the subject deeply and scientifically. In this treatise on ornamental design, the student will find a clue to the discovery of the source of an endless variety of beautiful forms and combinations of lines, in the application of certain fixed laws of harmonious proportion to the purposes of art. Mr Hay also exemplifies the application of his theory of linear harmony to the production of beautiful forms generally, testing its soundness by applying it to the human figure, and the purest creations of Greek art.

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Each part of this work is enriched by diagrams of great beauty, direct emanations of principle, and, consequently, presenting entirely new combinations of form. Had our space permitted, we should have made some extracts from this "Essay on Ornamental Design;" and we would have done so, because of the discriminating taste by which it is pervaded, and the forcible observations which it contains; but we cannot venture on the indulgence.

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From Blackwood's Magazine.

We have no doubt that when Mr Hay's Art-discovery is duly developed and taught, as it should be, in our schools, it will do more to improve the general taste than anything which has yet been devised.